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The scattering of harmonic elastic anti-plane shear waves by two collinear cracks in anisotropic material plane by using the non-local theory

Zhen-Gong Zhou · Lin-Zhi Wu · Biao Wang

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Abstract In this paper, the dynamic behavior of two collinear cracks in the anisotropic elasticity material plane subjected to the harmonic anti-plane shear waves is investigated by use of the nonlocal theory. To overcome the mathematical difficulties, a one-dimensional nonlocal kernel is used instead of a two-dimensional one for the anti-plane dynamic problem to obtain the stress field near the crack tips. By use of the Fourier transform, the problem can be solved with the help of a pair of triple integral equations, in which the unknown variable is the displacement on the crack surfaces. To solve the triple integral equations, the displacement on the crack surfaces is expanded in a series of Jacobi polynomials. Unlike the classical elasticity solutions, it is found that no stress singularity is present near crack tips. The nonlocal elasticity solutions yield a finite hoop stress at the crack tips, thus allowing us to using the maximum stress as a fracture criterion. The magnitude of the finite stress field not only depends on the crack length but also on the frequency of the incident waves and the lattice parameter of the materials.

Z.-G. Zhou (✉) · L.-Z. Wu · B. Wang
Center for composite materials, Harbin Institute of Technology, P.O. Box 1247, Harbin 150001, P.R. China
e-mail: zhouzhg@hit.edu.cn

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1 Introduction

As is commonly known, one of the principal postulates of the traditional mechanics of continuous media is the principle of the local action. This principle excludes the action at a distance, and attributes changes occurring at a point of the medium to thermoenergetic factors acting at the point. Of necessity then, the classical theory, by restricting the response of the continuum to strictly local actions, constitutes a so-called local theory. However, the application of classical elasticity to micromechanics leads to some physically unreasonable results. A singularity appearing in a stress field is a typical one. In fact, the stress at the crack tip is finite. As a result of this, beginning with Griffith, all fracture criteria in practice today are based on other considerations, e.g. energy, and the J-integral [1] and the strain gradient theory [2].

To overcome the stress singularity in the classical elastic fracture theory, Eringen [3–5] used nonlocal theory to discuss the stress near the tip of a sharp line crack in an isotropic elastic plate subject to uniform tension, shear and anti-plane shear, and the resulting solutions did not contain any stress singularities. This allows us to using the maximum stress as a fracture criterion. In [6], the basic theory of nonlocal elasticity was stated with emphasis on the difference between the nonlocal theory and classical continuum mechanics. Other results have been given by the application of nonlocal elasticity to the fields such as a dislocation near a crack [7, 8] and fracture mechanics problems [9, 10]. The results of those concrete problems that were solved display a rather remarkable agreement with experimental evidence. This can be used to predict the cohesive stress for various materials and the results close to those obtained in atomic lattice dynamics [11, 12]. Recently, some fracture problems [13–18] in the isotropic elastic material, the anisotropic elasticity material, and the piezoelectric material have been studied by use of nonlocal theory with a somewhat different method. In spite of these efforts, the understanding of the dynamic fracture process of anisotropic elastic materials is still limited due to the mathematical complexities. To our knowledge, the dynamic behavior of two collinear cracks in anisotropic material plane subjected to the harmonic anti-plane shear waves has not been studied by use of the nonlocal theory. Thus, the present work is an attempt to fill this information needed. Here, we just attempt to give a theoretical solution for this kind problem.

In the present paper, the dynamic behavior of two collinear cracks subjected to the harmonic anti-plane shear wave is investigated by use of nonlocal theory in anisotropic elastic material plane with Schmidt method [19, 20]. The Fourier transform is applied and a mixed boundary value problem is reduced to a pair of triple integral equations. To solve the triple integral equations, the displacement on the crack surfaces is expanded in a series of Jacobi polynomials. This process is quite different from those adopted in [1–12] as mentioned above. Numerical solutions are obtained for the stress field near the crack tip. Contrary to the previous results, it is found that the solution does not contain any stress singularities near the crack tip.

2 The crack model

It is assumed that there are two collinear symmetric cracks of length $1 - b$along the x-axis in an anisotropic material plane as shown in Fig. 1. The $2b$ is the distance between the two cracks (the solution

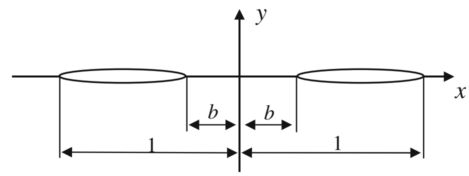

Fig. 1 Geometry and coordinate system for two collinear cracks

of two collinear cracks of length $d - b$ in anisotropic materials can easily be obtained by a simple change in the numerical values of the present paper for crack length $1 - b/d.d > b > 0$). In the present paper, it is also assumed that the propagation direction of the harmonic elastic anti-plane shear stress wave is vertical to the crack in anisotropic materials. Let ω be the circular frequency of the incident wave. $w_0(x, y, t)$ is the out-of-plane displacement. $\tau_{zk0}(x, y, t)(k = x, y)$ is the nonlocal anti-plane shear stress field. $\sigma_{zk0}(x, y, t)(k = x, y)$ is the local anti-plane shear stress field. As discussed in [21], because of the incident waves is the harmonic anti-plane shear stress wave, all field quantities of $w_0(x, y, t)$, $\tau_{zk0}(x, y, t)$, and $\sigma_{zk0}(x, y, t)$can be assumed to be of the forms as follows

$$[w_0(x, y, t),\ \tau_{zk0}(x, y, t),\ \sigma_{zk0}(x, y, t)], \\ = [w(x, y), \tau_{zk}(x, y),\ \sigma_{zk}(x, y)]e^{-i\omega t}. \tag{1}$$

In what follows, the time dependence of $e^{-i\omega t}$ will be suppressed but understood. $-\tau_0$ is a magnitude of the incident wave. As discussed in [3, 14, 21], the boundary conditions can be written as following (in this paper, we just consider the perturbation stress field):

$$\tau_{yz}(x, 0^+) = \tau_{yz}(x, 0^-) = -\tau_0, \\ b \le |x| \le 1,\ y = 0, \tag{2}$$

$$\tau_{yz}(x, 0^+) = \tau_{yz}(x, 0^-), w(x, 0^+) = w(x, 0^-) = 0, \\ |x| > 1, |x| < b,\ y = 0. \tag{3}$$

3 Basic equation of nonlocal theory

Basic equations of two-dimensional anti-plane shear of homogeneous, anisotropic, nonlocal elastic solid, with vanishing body force are

$$\frac{\partial \tau_{xz}(x, y)}{\partial x} + \frac{\partial \tau_{yz}(x, y)}{\partial y} = -\rho\omega^2 w(x, y), \tag{4}$$

$$\tau_{xz}(x, y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \alpha\left(\left|x' - x\right|, \left|y' - y\right|\right) \sigma_{xz}(x', y')\mathrm{d}x'\mathrm{d}y', \tag{5}$$

$$\tau_{yz}(x, y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \alpha\left(\left|x' - x\right|, \left|y' - y\right|\right) \sigma_{yz}(x', y')\mathrm{d}x'\mathrm{d}y', \tag{6}$$

where

$$\sigma_{xz}(x, y) = c_{55}\frac{\partial w(x, y)}{\partial x} + c_{45}\frac{\partial w(x, y)}{\partial y},$$
$$\sigma_{yz}(x, y) = c_{45}\frac{\partial w(x, y)}{\partial x} + c_{44}\frac{\partial w(x, y)}{\partial y}, \tag{7}$$

where $-\rho\omega^2 w(x, y)\mathrm{e}^{-i\omega t} = \rho\frac{\partial^2 w_0(x,y,t)}{\partial t^2} = \rho\frac{\partial^2 (w(x,y)\mathrm{e}^{-i\omega t})}{\partial t^2}$ in Eq. 4. c_{44}, c_{45}, and c_{55} are the material constants of the classical elasticity. ρ is the density of the materials. The only difference from the classical elasticity is in the stress constitutive equations (5) and (6). The stress $\tau_{xz}(x, y)$ and $\tau_{yz}(x, y)$ at a point (x, y) depend on the $\frac{\partial w(x,y)}{\partial x}$ and $\frac{\partial w(x,y)}{\partial y}$ at all points of the body. $\alpha(|x' - x|, |y' - y|)$ is known as the influence function and it is a function of the distance $d = \sqrt{(x' - x)^2 + (y' - y)^2}$. The expressions (7) are the classical Hooke's law.

4 The triple integral equation

Substitution of Eqs. 5 and 6 into Eq. 4 and using Green-Gauss theorem leads to

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \alpha\left(\left|x' - x\right|, \left|y' - y\right|\right)\left[c_{55}\frac{\partial^2 w(x', y')}{\partial x'^2} + 2c_{45}\frac{\partial^2 w(x', y')}{\partial x'\partial y'} + c_{44}\frac{\partial^2 w(x', y')}{\partial y'^2}\right]\mathrm{d}x'\mathrm{d}y' - \left[\int_{-1}^{-b} + \int_{b}^{1}\right]\alpha\left(\left|x' - x\right|, \left|y' - y\right|\right)\left[\sigma_{yz}(x', 0^+) - \sigma_{yz}(x', 0^-)\right]\mathrm{d}x' = -\rho\omega^2 w(x, y). \tag{8}$$

Here the surface integral may be dropped since the displacement field vanishes at infinity.

As mentioned in [3], it can be obtained that $[\sigma_{yz}(x, 0^+) - \sigma_{yz}(x, 0^-)] = 0$. What now remains is to solve the integrodifferential equation (8) for the function w. It is impossible to obtain a rigorous solution at the present stage. It seems obvious that in the solution of such a problem we encounter serious if not unsurmountable mathematical difficulties and will have to resort to an approximate procedure. In the given problem, as discussed in [22, 23], the nonlocal interaction in the y-direction can be ignored. In view of our assumptions, it can be given as

$$\alpha\left(\left|x' - x\right|, \left|y' - y\right|\right) = \alpha_0(\left|x' - x\right|)\delta(y' - y), \tag{9}$$

where $\alpha_0(\left|x' - x\right|) = \frac{1}{\sqrt{\pi}}(\beta/a)\exp[-(\beta/a)^2(x' - x)^2]$, $\delta(y' - y)$ a pulse function. β a constant and can be obtained by experiment, and a is the characteristic length. The characteristic length may be selected according to the range and sensitivity of the physical phenomena. For instance, for the perfect crystals, a may be taken as the lattice parameter. For granular materials, a may be considered to be the average granular distance and for fiber composites, the fiber distance, etc. In the present paper, a is taken as the lattice parameter.

From Eq. 8, we have

$$\int_{-\infty}^{\infty}\alpha_0(|x' - x|)\left[c_{55}\frac{\partial^2 w(x', y)}{\partial x'^2} + 2c_{45}\frac{\partial^2 w(x', y)}{\partial x'\partial y} + c_{44}\frac{\partial^2 w(x', y)}{\partial y^2}\right]\mathrm{d}x' = -\rho\omega.^2 w(x, y) \tag{10}$$

Introduce a coordinate transformation

$$\hat{x} = x' - ey, \quad \hat{y} = cy, \tag{11}$$

where $e = c_{45}/c_{44}$, $c = \mu/c_{44}$, $\mu = (c_{44}c_{55} - c_{45}^2)^{1/2}$. From Eq. 10, we have

$$\int_{-\infty}^{\infty}\alpha_0\left(\left|\hat{x} - u)\right|\right)\mu_0[\frac{\partial^2 w(\hat{x}, \hat{y})}{\partial \hat{x}^2} + \frac{\partial^2 w(\hat{x}, \hat{y})}{\partial \hat{y}^2}]\mathrm{d}\hat{x} = -\rho\omega^2 w(u, y'), \tag{12}$$

where $u = x - ey$ and $\mu_0 = (c_{44}c_{55} - c_{45}^2)/c_{44}$.

To solve the problem, the Fourier cosine transform of Eq. 12 with u can be given as follows:

$$\bar{\alpha}_0(|s|)\mu_0\left[-s^2\bar{w}(s, \hat{y}) + \frac{\partial^2\bar{w}(s, \hat{y})}{\partial \hat{y}^2}\right] = -\rho\omega^2\bar{w}(s, \hat{y}). \tag{13}$$

A superposed bar indicates the Fourier cosine transform through the paper.

From Eq. 9, we have

$$\bar{\alpha}_0(s) = \exp\left(-\frac{(sa)^2}{4\beta^2}\right). \tag{14}$$

Because of the symmetry, it suffices to consider the problem for $x \geq 0, |y| < \infty$. The solution of Eq. 13 can be given as follows

$$\begin{aligned} w(x,y) &= w(\hat{x},\hat{y}) \\ &= \begin{cases} \frac{2}{\pi}\int_0^\infty A(s)e^{-\gamma\hat{y}}\cos(s\hat{x})\mathrm{d}s, & \hat{y} \geq 0, \\ -\frac{2}{\pi}\int_0^\infty A(s)e^{\gamma\hat{y}}\cos(s\hat{x})\mathrm{d}s, & \hat{y} \leq 0, \end{cases} \\ &= \begin{cases} \frac{2}{\pi}\int_0^\infty A(s)e^{-c\gamma y}\cos(sx - esy)\mathrm{d}s, & y \geq 0, \\ -\frac{2}{\pi}\int_0^\infty A(s)e^{c\gamma y}\cos(sx - esy)\mathrm{d}s, & y \leq 0, \end{cases} \end{aligned} \tag{15}$$

where $\gamma = \sqrt{s^2 - \omega^2/c_1^2\bar{\alpha}_0}$, $c_1 = \sqrt{\mu_0/\rho}$.

It is easily verified from (4) and (10) that the relevant displacement and stress components in a physical anisotropic solid are related to those in the corresponding isotropic solid by

$$\sigma_{xz}(x,y) = (\mu/c_{44})\sigma_{\hat{x}\hat{z}}(\hat{x},\hat{y}) + (c_{45}/c_{44})\sigma_{\hat{y}\hat{z}}(\hat{x},\hat{y}), \tag{16}$$

$$\sigma_{yz}(x,y) = \sigma_{\hat{y}\hat{z}}(\hat{x},\hat{y}), \tag{17}$$

$$\sigma_{\hat{x}\hat{z}}(\hat{x},\hat{y}) = \mu\frac{\partial w(\hat{x},\hat{y})}{\partial\hat{x}}, \tag{18}$$

$$\sigma_{\hat{y}\hat{z}}(\hat{x},\hat{y}) = \mu\frac{\partial w(\hat{x},\hat{y})}{\partial\hat{y}}. \tag{19}$$

Substituting Eq. 13 into Eq. 3 and applying Eqs. 4, 18, and 19, it can be obtained

$$\begin{aligned} \tau_{yz}(x,y) &= -\frac{2\mu}{\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\alpha\left(|x'-x|,|y'-y|\right) \times\frac{\partial w(x',y')}{\partial y'}\mathrm{d}x'\mathrm{d}y' \\ &= -\frac{2\mu}{\pi}\left\{\int_0^\infty\left[\int_{-\infty}^{\infty}\alpha\left(|x'-x|,|y'-y|\right)\times\frac{\partial w(x',y')}{\partial y'}\mathrm{d}x'\right]\mathrm{d}y'\right. \\ &\quad\left.+\int_{-\infty}^{0}\left[\int_{-\infty}^{\infty}\alpha\left(|x'-x|,|y'-y|\right)\times\frac{\partial w(x',y')}{\partial y'}\mathrm{d}x'\right]\mathrm{d}y'\right\} \\ &= -\frac{2\mu}{\pi}\int_0^\infty\gamma\mathrm{e}^{-c\gamma y}A(s)\mathrm{d}s \\ &\quad\times\int_{-\infty}^{\infty}[\alpha_0\left(|x'-x|\right)\cos\left(sx'-esy\right) \\ &\quad+\alpha_0\left(|x'-x|\right)\cos(sx'+esy)]\mathrm{d}x'. \end{aligned} \tag{20}$$

Using Eqs. 16–18, it can be obtained

$$\begin{aligned} \tau_{xz}(x,y) &= -\frac{2\mu^2}{\pi c_{44}}\int_0^\infty s\mathrm{e}^{-c\gamma y}A(s)\mathrm{d}s \\ &\quad\times\int_{-\infty}^{\infty}\left[\alpha_0\left(|x'-x|\right)\sin(sx'-esy)\right. \\ &\quad\left.-\alpha_0\left(|x'-x|\right)\sin(sx'+esy)\right]\mathrm{d}x' \\ &\quad+\frac{c_{45}}{c_{44}}\tau_{yz}(x,y). \end{aligned} \tag{21}$$

From the relations [24],

$$\begin{aligned} &\int_{-\infty}^{\infty}\exp(-px'^2)\begin{Bmatrix}\sin\xi(x'+x)\\ \cos\xi(x'+x)\end{Bmatrix}\mathrm{d}x' \\ &= (\pi/p)^{1/2}\exp\left(-\frac{\xi^2}{4p}\right)\begin{Bmatrix}\sin(\xi x)\\ \cos(\xi x)\end{Bmatrix} \end{aligned} \tag{22}$$

the boundary conditions (6) and (7) can be expressed as:

$$\begin{aligned} \tau_{yz}(x,0) &= -\frac{4\mu}{\pi}\int_0^\infty\gamma\bar{\alpha}_0(s)A(s)\cos(sx)\mathrm{d}s \\ &= -\tau_0, \quad b \leq x \leq 1, \end{aligned} \tag{23}$$

$$\int_0^\infty A(s)\cos(sx)\mathrm{d}s = 0, \quad x > 1, 0 < x < b. \tag{24}$$

The stress τ_{xz} along the crack line can be expressed as:

$$\tau_{xz}(x,0) = \frac{c_{45}}{c_{44}}\tau_{yz}(x,0). \tag{25}$$

To determine the unknown function $A(s)$, the previous pair of triple integral equations (23) and (24) must be solved.

5 Solution of the triple integral equation

The only difference between the classical and nonlocal equations is in the influence function $\bar{\alpha}_0(s)$, it is logical to utilize the classical solution to convert the system Eqs. 23 and 24 to an integral equation of the second kind, which is generally better behaved. For the lattice parameter $a \to 0$, then $\bar{\alpha}_0(s)$ equals to a nonzero constant and Eqs. 23 and 24 reduce to a pair of triple integral equations for the same problem in classical elasticity. As discussed in [14–21], the triple integral equations (23) and (24) cannot be transformed into a Fredholm integral equation of the second kind, because $\bar{\alpha}_0(s)$ does not tend to a nonzero constant $C(C \neq 0)$for $s \to \infty$. Of course, the triple equations (23) and (24) can be considered to be a single integral equation of the first kind with discontinuous kernel. It is well-known in the literature that integral equations of the first kind are generally ill-posed in sense of Hadamard, i.e. small perturbations of the data can yield arbitrarily large changes in the solution. This makes the numerical solution of such equations quite difficult. To overcome the difficult, the Schmidt method [19, 20] is used to solve the triple integral equations (23) and (24). The displacement w on the crack surface can be represented by the following series:

$$w(x,0) = \sum_{n=0}^{\infty} b_n P_n^{(\frac{1}{2},\frac{1}{2})}\left(\frac{x-\frac{1+b}{2}}{\frac{1-b}{2}}\right) \times \left(1 - \frac{\left(x-\frac{1+b}{2}\right)^2}{\left(\frac{1-b}{2}\right)^2}\right)^{(1/2)}, \quad \text{for } b \le x \le 1 \text{ and } y = 0 \tag{26}$$

$$w(x,0) = 0, \quad \text{for } x > 1, 0 < x < b \quad \text{and } y = 0, \tag{27}$$

where b_n are unknown coefficients, $P_n^{(1/2,1/2)}(x)$ is a Jacobi polynomial [24]. The Fourier transform of Eqs. 26 and 27 are [25]

$$A(s) = \bar{w}(s,0) = \sum_{n=0}^{\infty} b_n F_n G_n(s) \frac{1}{s} J_{n+1}\left(s\frac{1-b}{2}\right) \tag{28}$$

where

$$F_n = 2\sqrt{\pi}\frac{\Gamma(n+1+\frac{1}{2})}{n!},$$

$$G_n(s) = \begin{cases} (-1)^{n/2}\cos\left(s\frac{1+b}{2}\right), & n = 0,2,4,6,\ldots \\ (-1)^{\frac{n+1}{2}}\sin\left(s\frac{1+b}{2}\right), & n = 1,3,5,7,\ldots \end{cases},$$

$\Gamma(x)$ and $J_n(x)$ are the Gamma and Bessel functions, respectively.

Substituting Eq. 28 into Eqs. 23 and 24, it can be shown that Eq. 24 is automatically satisfied. Equation (23) reduces to

$$\frac{4\mu}{\pi}\sum_{n=0}^{\infty} b_n F_n \int_0^{\infty} \frac{\gamma}{s}\bar{\alpha}_0(s) G_n(s) J_{n+1}\left(s\frac{1-b}{2}\right)\cos(sx)\mathrm{d}s = \tau_0 \quad b \le x \le 1 \quad \text{and } y = 0. \tag{29}$$

For a large s, the integrands of Eq. 29 almost all decrease exponentially. So the semi-infinite integral in Eq. 29 can be evaluated numerically. Thus Eq. 29 can be solved for coefficients b_n by the Schmidt method [19, 20]. Here, it was omitted. It can be seen in [13–20].

6 Numerical calculations and discussion

The coefficients b_n are known, so that the entire stress field can be obtained. However, in fracture mechanics, it is important to determine the stress τ_{yz} and τ_{xz} in the vicinity of the crack tips. τ_{yz} and τ_{xz} along the crack line can be expressed as:

$$\tau_{yz}(x,0) = -\frac{4\mu}{\pi}\sum_{n=0}^{\infty} b_n F_n \int_0^{\infty} \frac{\gamma}{s}\bar{\alpha}_0(s) G_n(s) J_{n+1} \times \left(s\frac{1-b}{2}\right)\cos(sx)\mathrm{d}s \quad \text{for } y = 0, \tag{30}$$

$$\tau_{xz}(x,0) = \frac{c_{45}}{c_{44}}\tau_{yz}(x,0) \quad \text{for } y = 0. \tag{31}$$

When the lattice parameter $a \neq 0$, the semi-infinite integration and the series in Eq. 30 are convergent for any variable x, it gives a finite stress all along $y = 0$, so there is no stress singularity at crack tips. At $b < x < 1$, $\tau_{yz}/(-\tau_0)$ is very close to unity, and for $x > 1$, $\tau_{yz}/(-\tau_0)$ possesses finite values diminishing from a finite value at $x = 1$ to zero at $x = \infty$. Since $a/[\beta(1-b)]{>}1/100$ represents a crack length of less than 100 atomic distances [5], and for such submicroscopic sizes, other serious questions arise regarding the interatomic arrangements and force laws, we do not pursue solutions valid at such small crack sizes. In the computation, the material constants are assumed to be $c_{44} = 2.77$ GPa, $c_{55} = 0.64$ GPa, and $c_{45} = 0.57$ GPa. From the works in [26–29], it can be seen that the Schmidt method is performed satisfactorily if the first ten terms of infinite series to Eq. 29 are retained. This

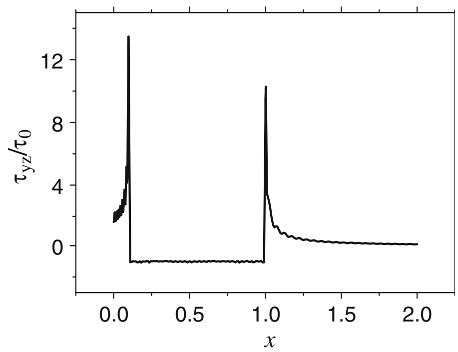

Fig. 2 The stress along the crack line versus x for $b = 0.1, a/\beta = 0.001$ and $\omega/c_1 = 0.2$

Schmidt method can be applicable to obliquely incident anti-plane waves. In this case, the $-\tau_0$ should be changed into$-\tau_0(\theta)$. It depends on the incident angle θ. It is independent of the variable x. The solving processes are the same as ones in the present paper. On the other hand, this method can be also extended to solve the problem of in-plane time-harmonic waves. However, the solving processes are more complex than the present paper.

The results are plotted in Figs. 2, 3, 4, 5, 6, 7, 8 and 9. The following observations are very significant:

(1) The maximum stress value does not occur at the crack tip, but slightly away from it as shown in Figs. 2, 3, 4, and 5. This phenomenon has been thoroughly substantiated by Eringen [30]. The maximum stress value is finite, thus allowing us to using the maximum stress value as a fracture criterion. The distance between the crack tip and the maximum stress point is very small, and it depends on the crack length, the material properties and the lattice parameter. Contrary to the classical elasticity solution, it is found that no stress singularity presents at the crack tip, and also the present results converge to the classical ones when far away from the crack tip as shown in Figs. 2, 3, 4, and 5. Simultaneously, for the nonlocal solution, the smaller the lattice parameter is, the more closer to the classical solution as shown in Figs. 2, 3, 4, and 5.

(2) The stress at the crack tip becomes infinite as the lattice parameter distance $a \to 0$. This is the classical continuum limit of square root singularity.

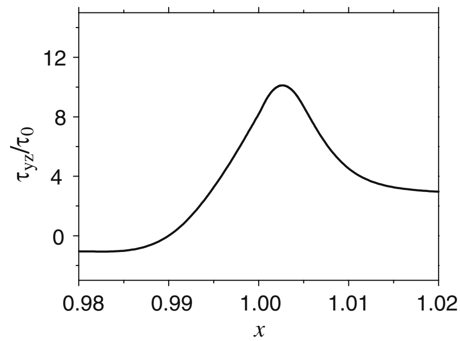

Fig. 3 The local enlarge graph of Fig. 2 near the outer tip of the right crack

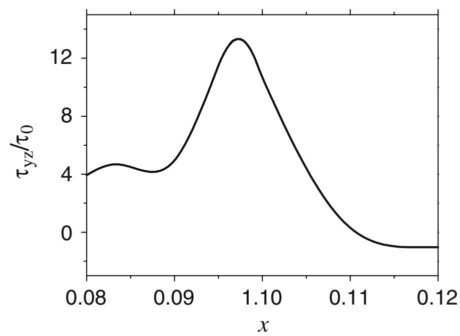

Fig. 4 The local enlarge graph of Fig. 2 near the inner tip of the right crack

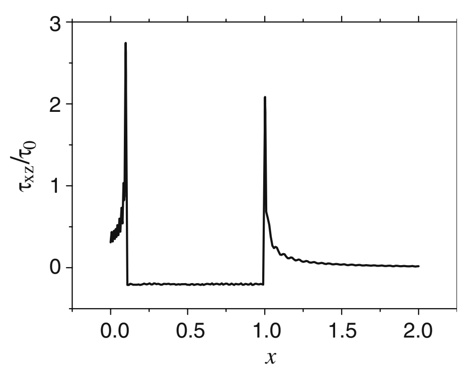

Fig. 5 The stress along the crack line versus x for $b = 0.1,\ a/\beta = 0.001$ and $\omega/c_1 = 0.2$

(3) The effect of the lattice parameter of the anisotropic composite materials on the stress field near the crack tip decreases with increase of the lattice parameter as shown in Fig. 6. This phenomenon is the same as one in [3–5].

(4) The stress values of τ_{yz} and τ_{xz} at the crack tips increases with increase of the crack length

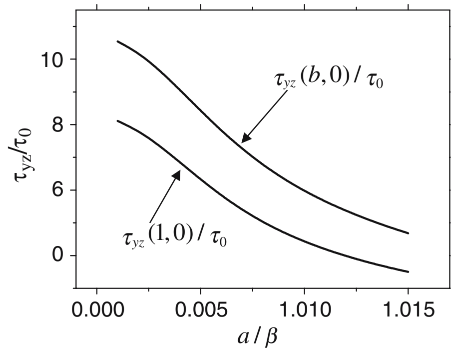

Fig. 6 The stress at the crack tips line versus a/β for $\omega/c_1 = 0.2$ and $b = 0.1$

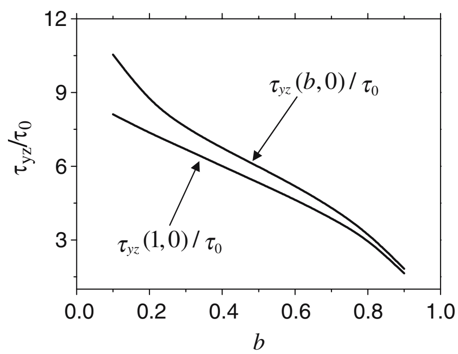

Fig. 7 The stress at the crack tips versus b for $\omega/c_1 = 0.2$, and $a/\beta = 0.001$

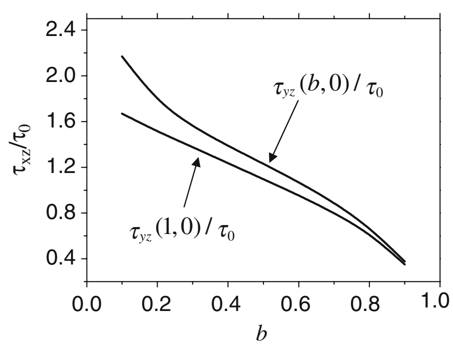

Fig. 8 The stress at the crack tips versus b for $\omega/c_1 = 0.2$, and $a/\beta = 0.001$

as shown in Figs. 7 and 8. Note this fact; experiments indicate that materials with smaller cracks are more resistant to fracture than those with larger cracks. This is similar with results of the classical theory. For the classical theory, the stress intensity factors increase with increase of the crack length. It can be also obtained that

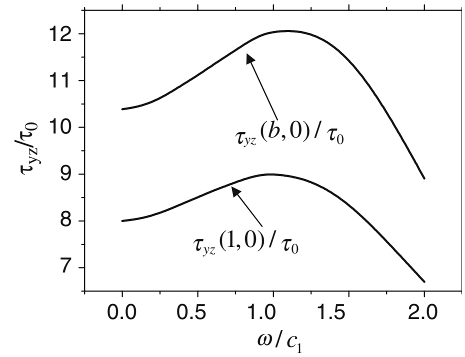

Fig. 9 The stress at the crack tips versus ω/c_1 for $b = 0.1$ and $a/\beta = 0.001$

the left tip's stress fields are greater than the right tip's ones for the right crack as shown in Figs. 7 and 8.

(5) The dynamic stress values of τ_{yz} at the crack tips tend to increase with the frequency reaching a peak for $\omega/c_1 \approx 1.1$ and then to decrease in magnitude as shown in Fig. 9. This conclusion is the same as in the classical fracture theory [31] for the stress intensity factor. From the results, it can be concluded that the stress fields near the crack tips can be deduced by adjusting the frequency of incident waves in engineering practices.

(6) The stress values of τ_{yz} at the crack tips do not depend on the material properties as shown in Eqs. 29 and 30. However, the stress of τ_{yz} depends on the crack length, on the frequency of the incident waves and the lattice parameter of the materials. This is the same as the anti-plane shear fracture problem in the isotropic homogeneous materials.

(7) The stress values of τ_{xz} depend on the material properties, on the crack length, the frequency of the incident waves and the lattice parameter of the materials as shown in Eq. 31. The variations of the stresses τ_{yz} and τ_{xz} have a same tendency with the crack length, the frequency of the incident waves or the lattice parameter of the materials as shown in Eq. 31. The variations of the stress τ_{xz} with the crack length, the frequency of the incident waves or the lattice parameter of the materials can be obtained through Eq. 31 from the present results. Here, they are omitted. However, the

amplitude values of τ_{xz} are different. The stress of τ_{yz} is larger than the stress of τ_{xz}.

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