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ANALYTICAL SOLUTIONS OF THERMAL STRESS DISTRIBUTION IN PLASTIC ENCAPSULATED INTEGRATED CIRCUIT PACKAGES *

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Abstract: *Due to the mismatch in the coefficients of thermal expansion of slicon chip and the surrounding plastic encapsulation materials, the induced thermal stress is the main cause for die and encapsulant rupture. The corner geometry is simplified as the semi infinite wedge. Then the two-dimensional thermal stress distribution around the corner was obtained explicitly. Based on the stress calculation, the strain energy density factor criterion is used to evaluate the strength of the structure, which can not only give the critical condition for the stresses, but also determine the direction of fracture initiation around the corner.*

Key words: electronic package structure; thermal stress; analytical solution

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Introduction

Electronic packages are complex composite structure. They have myriad corners and interfaces that can act as potential failure sites during field operation. At present time, plastic encapsulated integrated circuits (ICs) are widely used in engineering. A plastic encapsulated IC package consists of a silicon die, die attachment, passivation, wire interconnects, a lead frame, and plastic molding compound. Thermal excursion during package qualification from thermal cycling/thermal shock, or power cycling during normal operation can set up thermomechanical stresses due to the thermal expansion coefficient mismatch of different materials. Such stresses can be sufficient severe to induce microcracking around the corners, and lead ultimately to device failure (Bar-Cohen, 1992; Nguyen, *et al*., 1995; Holalkere, *et al*., 1997; Lau and Pao, 1997)[1-4].

To calculate the stress field and evaluate reliability behavior of the electronic packages, the finite element method has been widely used in the electronic industry and has shown some success (Pendse and Demmin, 1990; Nguyen, *et al*., 1993; Shook, *et al*., 1997; Zhu, *et al*.,

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1997, Clark, *et al*., 1993)[5-9]. Since the thermal stress is singular near the sharp corner of the electronic packages, the stress field obtained by the finite element analysis around the corner is mesh-dependent (Hu, 1995)[10]. Thus the ordinary analysis of the finite element method is quite unreliable for the stress concentration calculation at the corner. Another key unsolved issue is strength evaluation method and criterion. Since the stress is singular near the corners, all the failure criterions based on maximum stress, etc. are not suitable for strength evaluation. Some other strength evaluation methods are based on the singularity parameter (Hattori, *et al*., 1989)[11], or average stress in a finite local zone were proposed, but have not been widely adopted in practical design evaluation.

In this paper, it is found that under some reasonable assumptions, the corner geometry can be simplified as the semi-infinite wedge. Then the two-dimensional thermal stress distribution around the corner can be obtained explicitly. Based on the stress calculation, the strain energy density factor criterion is used to evaluate the strength of the structure that can not only give the critical condition for the stresses, but also determine the direction of fracture initiation around the corner.

1 Analytical Solutions of Thermal Stresses Around a Corner

As we have said, there are various corners in electronic packages. In this paper, we consider the die packaged in epoxy molding compound as an illustration (Fig. 1).

Fig. 1 Die encapsulated in epoxy molding compound

Since the silicon die has a much smaller thermal expansion coefficient than the surrounding epoxy molding compound, the thermal stresses will be produced along the interface during the cooling or heating process. If we are only interested in the thermal stress distribution around one corner, we can use the following wedge model (Fig. 2) to derive it explicitly. For square die, the normal thermal stress along the interface is assumed to be p_0 for $0 \leqslant r \leqslant a$, where a is the size of the die, and the stress p_0 can be determined through the continuity condition of the displacement along the interface. The epoxy molding compound is assumed to extend infinitely. Thus under the action of the stress p_0, the stress distribution both in the epoxy molding compound and inside the die can be solved separately. In this paper, we only show the solution process in the epoxy molding compound, since it is comparatively easy to obtain the analytical solution for the square die. In polar coordinates, stresses are connected by the following equilibrium equations:

$$\frac{\partial(r\sigma_r)}{\partial r} - \sigma_\theta + \frac{\partial \tau_{r\theta}}{\partial \theta} = 0, \qquad \frac{\partial(r\tau_{r\theta})}{\partial r} + \tau_{r\theta} + \frac{\partial \sigma_\theta}{\partial \theta} = 0. \tag{1}$$

Consider the isotropic epoxy molding compound. Then, stresses are connected with corresponding strain by the Hooke's law, i.e.,

$$\begin{cases} \varepsilon_r = \dfrac{1}{E}(\sigma_r - \nu\sigma_\theta), \quad \varepsilon_\theta = \dfrac{1}{E}(\sigma_\theta - \nu\sigma_r), \\ \varepsilon_{r\theta} = \dfrac{2(1+\nu)}{E}\tau_{r\theta}, \end{cases} \tag{2}$$

which is the relation for plane stress problem, where E and ν are Young's modulus and Poisson's ratio of the material. The strains, in turn, are expressed through displacements by geometric equations

$$\begin{cases} \varepsilon_r = \dfrac{\partial u_r}{\partial r}, \\ \varepsilon_\theta = \dfrac{1}{r}\dfrac{\partial u_\theta}{\partial \theta} + \dfrac{u_r}{r}, \\ \varepsilon_{r\theta} = \dfrac{1}{r}\dfrac{\partial u_r}{\partial \theta} + \dfrac{\partial u_\theta}{\partial r} - \dfrac{u_r}{r}, \end{cases} \tag{3}$$

where u_r and u_θ are the components of the displacements in the radial and tangential directions, respectively.

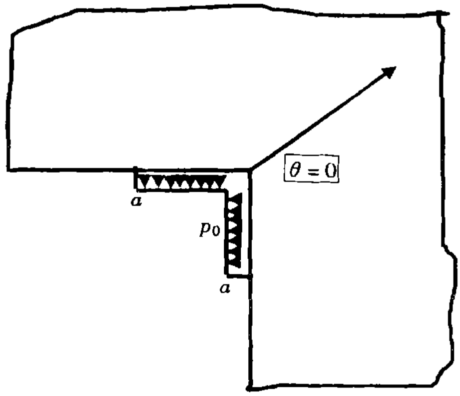

Fig. 2 A infinite wedge under the action of p_0

Eliminating displacements we can write the following compatibility equation:

$$\frac{\partial^2 \varepsilon_\theta}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 \varepsilon_r}{\partial \theta^2} + \frac{2}{r}\frac{\partial \varepsilon_\theta}{\partial r} - \frac{1}{r}\frac{\partial \varepsilon_r}{\partial r} = 2\left(\frac{1}{r}\frac{\partial^2 \varepsilon_{r\theta}}{\partial r\,\partial \theta} + \frac{1}{r^2}\frac{\partial \varepsilon_{r\theta}}{\partial \theta}\right). \tag{4}$$

Equations (1), (2), (3) and (4) form a complete set of equations for the plane problem of the theory of elasticity in polar coordinates.

Introducing the stress function, $\varphi(r, \theta)$ in accordance with the following relations:

$$\sigma_r = \frac{1}{r}\frac{\partial \varphi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \varphi}{\partial \theta^2}, \qquad \sigma_\theta = \frac{\partial^2 \varphi}{\partial r^2}, \qquad \tau_{r\theta} = -\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \varphi}{\partial \theta}\right). \tag{5}$$

We obtain for this stress function the biharmonic equation

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right)\left(\frac{\partial^2 \varphi}{\partial r^2} + \frac{1}{r}\frac{\partial \varphi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \varphi}{\partial \theta^2}\right) = 0. \tag{6}$$

To reflect the mismatch of the thermal expansion between the die and the surrounding epoxy molding compound, on finite segments of the contact lateral faces of the wedge $\theta = \pm 3\pi/4$, $0 \leqslant r \leqslant a$, it is subjected to a normal loading that is symmetric with respect to the axis of the wedge for square die, and the tangential stress can be set to zero on the faces of the wedge. Thus one has the boundary conditions as follows:

$$\begin{cases} \sigma_\theta\left(r, \dfrac{3}{4}\pi\right) = \sigma_\theta\left(r, -\dfrac{3}{4}\pi\right) = \begin{cases} p_0 & 0 \leqslant r \leqslant a, \\ 0 & \text{others}; \end{cases} \\ \tau_{r\theta}\left(r, \dfrac{3}{4}\pi\right) = \tau_{r\theta}\left(r, -\dfrac{3}{4}\pi\right) = 0 \qquad 0 \leqslant r < \infty. \end{cases} \tag{7}$$

To get a solution of the problem we apply the Mellin integral transformation in the variable r. As is well-known, the Mellin transform of a function $f(r)$ and its inverse are given by the relations

$$\mathcal{F}(s) = \int_0^\infty f(r) r^{s-1} \mathrm{d}r, \qquad f(r) = \frac{1}{2\pi \mathrm{i}}\int_{c-\mathrm{i}\infty}^{c+\mathrm{i}\infty} \mathcal{F}(s) r^{-s} \mathrm{d}s. \tag{8}$$

Applying the Mellin transformation to the biharmonic equation (6), we get for the transform, $\varphi(s,\theta)$, of the stress function the ordinary differential equation

$$\left[\frac{d^2}{d\theta^2}+s^2\right]\left[\frac{d^2}{d\theta^2}+(s+2)^2\right]\varphi(s,\theta)=0. \tag{9}$$

The stress field sought is symmetric with respect to the axis $\theta=0$, thus the stress function must be also so. The general symmetric solution of Eq.(9) has the form

$$\varphi(s,\theta)=C(s)\cos(s\theta)+D(s)\cos(s+2)\theta. \tag{10}$$

Substituting Eq.(5) into Eq.(7) gives the boundary condition for the stresss function

$$\frac{\partial^2\varphi(r,\pm 3\pi/4)}{\partial r^2}=p_0,\qquad \frac{1}{\partial r}\left[\frac{1}{r}\frac{\partial\varphi(r,\pm 3\pi/4)}{\partial\theta}\right]=0. \tag{11}$$

Multiplying Eq.(11) by r^2, and applying the Mellin transformation to these conditions we get the boundary conditions for the function $\varphi(s,\theta)$ as follows:

$$\varphi\left(s,\pm\frac{3}{4}\pi\right)=\frac{\bar{p}(s)}{s(s+1)},\qquad \frac{d\varphi(s,\pm 3\pi/4)}{d\theta}=0, \tag{12}$$

where

$$\bar{p}(s)=\int_0^a p_0 r^{s+1}dr. \tag{13}$$

Substituting solution (10) into the boundary condition (12) gives the constants $C(s)$ and $D(s)$ as follows:

$$\begin{cases} C(s)=-\dfrac{(s+2)\sin((s+2)(3\pi/4))}{s(s+1)[s+1-\sin((3\pi/2)(s+1))]}\bar{p}(s), \\ D(s)=\dfrac{\sin(3\pi s/4)}{(s+1)[s+1-\sin((3\pi/2)(s+1))]}\bar{p}(s). \end{cases} \tag{14}$$

Substituting the results into the inversion formula we obtain the solution for the stress function in the form

$$\begin{aligned}\varphi(r,\theta)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}&\frac{\bar{p}(s)r^{-s}}{s(s+1)[s+1-\sin((3\pi/2)(s+1))]}\times\\ &[s\sin(3\pi s/4)\cos((s+2)\theta)-\\ &(s+2)\sin((3\pi/4)(s+2))\cos(s\theta)]\,ds.\end{aligned} \tag{15}$$

Substituting the stress function (15) into Eq. (5) yields the components of the stess tensor in the form

$$\begin{aligned}\sigma_\theta(r,\theta)=\frac{\partial^2\varphi(r,\theta)}{\partial r^2}=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}&\frac{\bar{p}(s)r^{-s-2}}{s+1-\sin((3\pi/2)(s+1))}\times\\ &[s\sin(3\pi s/4)\cos((s+2)\theta)-(s+2)\sin((3\pi/4)(s+2))\cos(s\theta)]\,ds,\end{aligned} \tag{16}$$

$$\begin{aligned}\sigma_r(r,\theta)=\frac{1}{r}\frac{\partial\varphi(r,\theta)}{\partial r}+\frac{1}{r^2}\frac{\partial^2\varphi(r,\theta)}{\partial\theta^2}=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}&\frac{\bar{p}(s)r^{-s-2}}{s+1-\sin((3\pi/2)(s+1))}\times\\ &[(s+2)\sin((3\pi/4)(s+2))\cos(s\theta)-(s+4)\sin(3\pi s/4)\cos((s+2)\theta)]\,ds,\end{aligned} \tag{17}$$

$$\tau_{r\theta}(r,\theta) = -\frac{1}{\partial r}\left[\frac{1}{r}\frac{\partial\varphi(r,\theta)}{\partial\theta}\right] = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{\bar{p}(s)r^{-s-2}(s+2)}{s+1-\sin((3\pi/2)(s+1))}\times$$
$$[\sin((3\pi/4)(s+2))\sin(s\theta) - \sin(3\pi s/4)\sin((s+2)\theta)]\,\mathrm{d}s. \tag{18}$$

It is evident that the integrands in Eqs.(16), (17) and (18) are meromorphic functions of the complex variable s whose poles correspond to the roots of the following function:

$$s+1-\sin((3\pi/2)(s+1)) = 0. \tag{19}$$

It can be easily found that the approximate roots of Eq.(19) are as follows:

$$s_1 = -1.544\,48, \quad s_2 = -1, \quad s_3 = -0.455\,52. \tag{20}$$

The solutions of Eq. (19) are shown in Fig.3.

To calculate the integrals in Eqs.(16), (17) and (18), we must close the contour of integration by adding it to the line Re $(s) = c$ a semicircle of large radius on the left. According to the residue theorem, the required integrals can be expressed in terms of the sum of the residues at the poles contained in the contour obtained since the integrals along the semicircle are zero. Furthermore, to get a unique solution one imposes the additional regularity requirement of boundedness of the resulting force on any radial cut of the wedge:

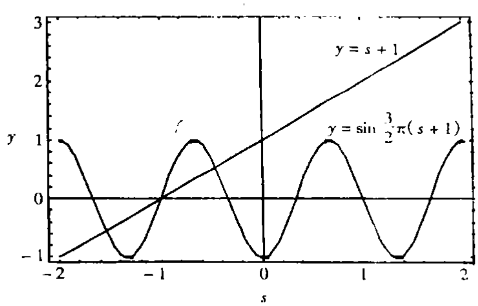

Fig.3 Graphical solution of Eq.(19)

$$\begin{cases} \int_0^\infty \sigma_\theta(r,\theta)\mathrm{d}r < \infty, \\ \int_0^\infty \tau_{r\theta}(r,\theta)\mathrm{d}r < \infty. \end{cases} \tag{21}$$

Therefore Re $(s) = c$ can be chosen so that the closed contour only contains the pole $s_1 = 1.544\,48$. The solutions for the stress fields become

$$\begin{cases} \sigma_\theta(r,\theta) = \bar{p}(s_1)r^{-s_1-2}\left[s_1\sin\left(\frac{3}{4}\pi s_1\right)\cos((s_1+2)\theta) - \right. \\ \qquad \left.(s_1+2)\sin\left(\frac{3}{4}\pi(s_1+2)\right)\cos(s_1\theta)\right], \\ \sigma_r(r,\theta) = \bar{p}(s_1)r^{-s_1-2}\left[(s_1+2)\sin\left(\frac{3}{4}\pi(s_1+2)\right)\cos(s_1\theta) - \right. \\ \qquad \left.(s_1+4)\sin\left(\frac{3}{4}\pi s_1\right)\cos((s_1+2)\theta)\right], \\ \tau_{r\theta}(r,\theta) = \bar{p}(s_1)r^{-s_1-2}(s_1+2)\times \\ \qquad \left[\sin\left(\frac{3}{4}\pi(s_1+2)\right)\sin(s_1\theta) - \sin\left(\frac{3}{4}\pi s_1\right)\sin((s_1+2)\theta)\right]. \end{cases} \tag{22}$$

From Eq.(22), it can be found that the stress field at the corner is singular with singularity exponent $\lambda = -s_1 - 2 = -0.455\,52$.

The components of the displacements in the radial and tangential directions can be derived from Eqs. (2) and (3) as follows:

$$u_r = \int \varepsilon_r \mathrm{d}r = \frac{1}{E}\int(\sigma_r - \gamma\sigma_\theta)\mathrm{d}r = \frac{\bar{p}(s_1)r^{-s_1-1}}{E(s_1+1)}\times$$
$$\left[(4+s_1+\gamma s_1)\sin\left(\frac{3}{4}\pi s_1\right)\cos((s_1+2)\theta)-\right.$$
$$\left.(1+\gamma)(s_1+2)\sin\left(\frac{3}{4}\pi(s_1+2)\right)\cos(s_1\theta)\right]+u_0(\theta), \tag{23}$$

where $u_0(\theta)$ is only a function of θ.

$$u_\theta = \int(r\varepsilon_\theta - u_r)\mathrm{d}\theta = \frac{1}{E}\int(\sigma_\theta - \gamma\sigma_r)r\mathrm{d}\theta - \int u_r\mathrm{d}\theta =$$
$$\frac{\bar{p}(s_1)^{-s_1-1}}{E}\left[\left(\frac{s_1+\gamma s_1+4\gamma}{s_1+2}-\frac{4+s_1+\gamma s_1}{(s_1+1)(s_1+2)}\right)\times\right.$$
$$\sin\left(\frac{3}{4}\pi s_1\right)\sin((s_1+2)\theta)+\frac{(1+\gamma)(s_1+2)}{s_1}\left(\frac{1}{s_1+1}-1\right)\times$$
$$\left.\sin\left(\frac{3}{4}\pi(s_1+2)\right)\sin(s_1\theta)\right]-\int u_0(\theta)\mathrm{d}\theta+v(r), \tag{24}$$

where $v(r)$ is a only function of r.

To determine the functions $u_0(\theta)$, $v(r)$, substituting the displacement components into the shear strain expression of Eq. (3),

$$\varepsilon_{r\theta} = \frac{2(1+\gamma)}{E}\sigma_{r\theta} = \frac{1}{r}\frac{\partial u_r}{\partial\theta}+\frac{\partial u_\theta}{\partial r}-\frac{u_\theta}{r}, \tag{25}$$

one obtains

$$\frac{1}{r}\frac{\partial u_0(\theta)}{\partial\theta}+\frac{\partial v(r)}{\partial r}+\frac{1}{r}\int u_0(\theta)\mathrm{d}\theta-\frac{1}{r}v(r)=0, \tag{26}$$

from which

$$v(r)=Fr, \qquad u_0(\theta)=H\sin\theta+K\cos\theta, \tag{27}$$

where F, H, and K are constants to be determined from the conditions of constraint. Since the axis $\theta=0$ is symmetric axis, one obtains

$$u_\theta=0, \qquad \frac{\partial u_\theta}{\partial r}=0, \qquad \text{for } \theta=0 \tag{28}$$

from which, it follows that $F=H=0$. For silicon chip encapsulated in IC plastic packages, one can assume that the displacement of EMC is restricted at the outside boundary, therefore

$$u_r=0 \qquad \text{for } \theta=0,\ r=R, \tag{29}$$

where R is the radial distance from the die corner to the outside corner of EMC. Thus, the constant K can be determined as follows:

$$K=-\frac{\bar{p}(s_1)R^{-s_1-1}}{E(s_1+1)}\left[(4+s_1+\gamma s_1)\sin\left(\frac{3}{4}\pi s_1\right)-\right.$$

$$(1+\gamma)(s_1+2)\sin\left(\frac{3}{4}\pi(s_1+2)\right)\Big]. \tag{30}$$

As we said have before, the acting force p_0 on the face of the wedge is created by the misfitting between the thermal expansion coefficient between the die and the epoxy molding compound. Since the elastic modulus of the die is much higher than that of the epoxy molding compound, the elastic displacement in the die can be considered zero. Therefore the displacement of EMC in the tangential direction along $\theta = \pm 3\pi/4$ can be written as

$$u_\theta\left(\pm\frac{3}{4}\pi\right) = \frac{1}{2}(\alpha_1 - \alpha_2)\Delta T a, \tag{31}$$

where α_1, α_2 are thermal expansion coefficients of the die and EMC, respectively, ΔT is the temperature difference, and a is the size of the die. From Eq. (24), it is found that the displacement in the tangential direction is not constant even if the applied force on the face is constant. To determine the interacting force p_0 between the die and EMC, we simply take the average displacement in the tangential direction for $0 \leqslant r \leqslant a$ as the thermal misfitting displacement, i.e., Eq. (31). Therefore, one derives an equation to determine p_0,

$$\frac{1}{a}\int_0^a u_\theta \,\mathrm{d}r = \frac{1}{2}(\alpha_1 - \alpha_2)\Delta T a. \tag{32}$$

Through Eq. (32), one can derive the acting thermal stress p_0 between the die and EMC as follows:

$$\begin{aligned} p_0 = \frac{1}{2}\Bigg\{ & \frac{-1}{s_1(s_1+2)}\left[\frac{s_1+\gamma s_1+4\gamma}{s_1+2} - \frac{4+s_1+\gamma s_1}{(s_1+1)(s_1+2)} + \right. \\ & \left.\frac{(1+\gamma)(s_1+2)}{s_1}\left(\frac{1}{s_1+1}-1\right)\right]\sin\left(\frac{3}{4}\pi(s_1+2)\right)\sin\left(s_1\frac{3}{4}\pi\right) - \\ & \frac{1}{(s_1+1)(s_1+2)}\left(\frac{a}{R}\right)^{s_1+1}\left[(1+\gamma)(s_1+2)\sin\left(\frac{3}{4}\pi(s_1+2)\right) - \right. \\ & \left.(4+s_1+\gamma s_1)\sin\left(\frac{3}{4}\pi s_1\right)\right]\sin\left(\frac{3}{4}\pi\right)\Bigg\}^{-1} E(\alpha_1 - \alpha_2)\Delta T. \end{aligned} \tag{33}$$

Substituting the values $s_1 = 1.544\,48$, $\gamma = 0.34$ into Eq. (33), one obtains

$$p_0 \approx \frac{1}{2}\left[2.820\,2 + 4.155 \times \left(\frac{a}{R}\right)^{-0.544\,48}\right]^{-1} E(\alpha_1 - \alpha_2)\Delta T. \tag{34}$$

One should bear in mind that R is the distance from the die corner to the point where the radial displacement is zero along the axis $\theta = 0$. As a limiting case, if we fix the point where $r = R = 0$, Eq. (34) gives the maximum thermal stress along the face, i.e.,

$$p_0 \approx 0.177\,3\, E(\alpha_1 - \alpha_2)\Delta T. \tag{35}$$

And if $R \to \infty$, the interacting stress approaches zero. That is true since for the wedge problem we consider, it can satisfy the displacement requirement through rigid movement. For the electronic packages we considered, due to the symmetric requirement of the die and surrounding EMC, the perpendicular displacements along the axis X, and Y that pass the center of the die are zero. Thus one should put a constraint on the displacement.

2 Strength Evaluation

Since the stresses around the sharp corner is singular with $r^{-0.45552}$, we cannot use the criterion based on the maximum stress or strain to evaluate the strength of the structure around the corner. And since there is no crack there, we cannot also use the fracture mechanics to determine when the corner will break. In this paper, we try to adapt the strain energy density criterion proposed by Sih (1973a, b), (1974)[12-14] in fracture mechanics to evaluate the strength of the structure. Using the SED criterion, we can not only determine when the material will fail, but also determine the direction of cracking around the corner. The strain energy density function can be expressed in the form

$$\frac{\mathrm{d}W}{\mathrm{d}V}=\frac{S}{r^{0.9111}}, \tag{36}$$

where W is the strain energy, S is the strain energy density factor. There are three basic hypotheses of the strain energy density criterion (Gdoutos, 1990)[15]:

Hypothesis 1 The location of fracture coincides with the location of minimum strain energy density, and yielding with maximum strain energy density;

Hypothesis 2 Failure by fracture or yielding occurs when $(\mathrm{d}W/\mathrm{d}V)_{\min}$ or $(\mathrm{d}W/\mathrm{d}V)_{\max}$ reach their respective critical values;

Hypothesis 3 The amount of incremental growth r_1, r_2, …, r_j, …, r_c is governed by

$$\left(\frac{\mathrm{d}W}{\mathrm{d}V}\right)_c=\frac{S_1}{r_1}=\frac{S_2}{r_2}=\cdots=\frac{S_j}{r_j}=\cdots=\frac{S_c}{r_c}, \tag{37}$$

there is unstable fracture or yielding when the critical ligament size r_c is reached.

Hypothesis 1 means that the relative local minimum of $\mathrm{d}W/dV$ corresponds to large volume change and identified with the region dominated by macrodilatation leading to fracture, while the relative local maximum of $\mathrm{d}W/\mathrm{d}V$ corresponds to large shape change and identified with the region dominated by macrodistortion leading to yielding. Therefore one should determine whether to use the minimum of $\mathrm{d}W/\mathrm{d}V$, or the maximum of $\mathrm{d}W/\mathrm{d}V$ according to the failure mechanism. Hypothesis 3 is for damage propagation problem. Using the stress field obtained above, Eq. (22), we can derive the strain energy density factor S as follows:

$$\begin{aligned}
S = {} & r^{0.9111}\frac{1}{4G}\left[\frac{k+1}{4}(\sigma_r+\sigma_\theta)^2-2(\sigma_r\sigma_\theta-\tau_{r\theta}^2)\right]= \\
& \frac{p_0^2a^{2(s_1+2)}}{(s_1+2)^2G}\left\{(k+1)\sin^2\left(\frac{3}{4}\pi s_1\right)\cos^2((s_1+2)\theta)-\right. \\
& \frac{1}{2}\left[(s_1+2)\sin\left(\frac{3}{4}\pi(s_1+2)\right)\cos(s_1\theta)-\right. \\
& \left.(s_1+4)\sin\left(\frac{3}{4}\pi s_1\right)\cos((s_1+2)\theta)\right]\left[s_1\sin\left(\frac{3}{4}\pi s_1\right)\cos((s_1+2)\theta)-\right. \\
& \left.(s_1+2)\sin\left(\frac{3}{4}\pi(s_1+2)\right)\cos(s_1\theta)\right]+ \\
& \left.\frac{1}{2}(s_1+2)^2\left[\sin\left(\frac{3}{4}\pi(s_1+2)\right)\sin(s_1\theta)-\sin\left(\frac{3}{4}\pi s_1\right)\sin((s_1+2)\theta)\right]^2\right\},
\end{aligned} \tag{38}$$

where G is the shear modulus such that $G = E/[2(1+\gamma)]$, $k = 3 - 4\gamma$ for plane strain, and $k = (3-\gamma)/(1+\gamma)$ for generalized plane stress.

The first hypothesis of SED criterion can be expressed mathematically by the relations

$$\frac{\partial S}{\partial \theta} = 0, \qquad \frac{\partial^2 S}{\partial \theta^2} > 0. \tag{39}$$

The cracking direction at the corner can be determined by Eq. (39) as $\theta_c = 0$.

Crack initiation occurs when

$$S(\theta_c) = S_c, \tag{40}$$

where S_c is the critical value of the strain energy density factor which is a material constant. Therefore one obtains

$$S(\theta_c) = \frac{p_0^2 a^{2(s_1+2)}}{(s_1+2)^2 G}\left\{(k+1)\sin^2\left(\frac{3}{4}\pi s_1\right) - \frac{1}{2}\left[(s_1+2)\sin\left(\frac{3}{4}\pi(s_1+2)\right) - (s_1+4)\sin\left(\frac{3}{4}\pi s_1\right)\right]\left[s_1\sin\left(\frac{3}{4}\pi s_1\right) - (s_1+2)\sin\left(\frac{3}{4}\pi(s_1+2)\right)\right]\right\} = S_c. \tag{41}$$

From Eq. (41), one can easily find that when increasing the size of the die, the critical thermal load which induces the failure of the corner is decreasing, i. e., under the same operation condition, the larger the die, the easier for the crack initiation around the corner.

3 Concluding Remarks

Electronic packages are complex composite structure. They have myriad corners and interfaces which can act as potential failure sites during field operation. In this paper, it is found that under some reasonable assumptions, the corner geometry can be simplified as the semi-infinite wedge. Then the two-dimensional thermal stress distribution around the corner can be obtained explicitly. Although the paper focus on the special symmetric structure, i.e., a square die encapsulated in epoxy molding compound, the analysis can be extended to more complex structures with various corners. It is found that the stresses around the corner have $r^{-0.45552}$ singularity for such case. The interacting thermal stress between the die and EMC is also obtained based on the continuity condition of the displacement. Based on the stress calculation, the strain energy density factor criterion is used to evaluate the strength of the structure which can not only give the critical condition for the stresses, but also determine the direction of fracture initiation around the corner. It is found that when increasing the size of the die, the critical thermal load which induces the failure of the corner is decreasing in proportion to $a^{-0.45552}$.

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