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ON THE CALCULATION OF ENERGY RELEASE RATE FOR VISCOELASTIC CRACKED LAMINATES *

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Abstract: *The energy release rate (ERR) of crack growth as the energy change at the same time t between the two states of the structure is redefined, one is with crack length a under the loading $\sigma(t)$, the other is the state with crack length $a + \Delta a$ under the same loading condition. Thus the defined energy release rate corresponds to the released energy when a crack grows from a to $a + \Delta a$ in an infinitesimal time. It is found that under a given loading history, the ERR is a function of time, and its maximum value should correspond with the critical state for delamination to propagate. Following William's work, the explicit expressions of ERR for DCB experimental configurations to measure the interfacial fracture toughness have been obtained through the classical beam assumption.*

Key words: composite laminate; viscoelastic model; analysis delamination

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Introduction

Generally speaking, a crack in viscoelastic materials will grow in some unknown speed even if the applied load is quasistatic, thus the main difficulty is induced in derving the energy release rate.

Knowledge of the condition governing the delamination in composite laminates with viscoelastic layers is of paramount importance in practical applications. For example, in plastic encapsulated IC packages, the interfacial delamination between the Silicon die and viscoelastic epoxy molding compound under the thermal loading is the main failure mode of the structure[1,2]. The phenomenon of crack growth in a general, linearly viscoelastic and isotropic material has been extensively studied during the past two decades[2-10]. Many theories proceed directly from an extension of Griffith criterion of overall energy conservation for a system involving energy dissipation. Generally speaking, for a problem with moving boundary such as crack growth, the

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correspondence principle that was originally proposed by Alfrey[11] to establish the relationship between the elasticity and viscoelasticity in quasi-static problem is not valid. Thus it restricts people to derive the stress and strain field. Especially for interfacial crack between viscoelastic layers, the analyses are much more difficult than their elastic counterparts. It seems that currently the only available results are those for Mode III loading[12,13].

The main difficulty in deriving the energy release rate of crack propagation in viscoelastic materials is due to that the crack will grow in some unknown speed even if the applied load is quasistatic. In this paper, we redefine the energy release rate (ERR) of crack growth as the energy change at the same time t between the two states of the structure, one is with crack length a under the loading $\sigma(t)$; The other is the state with crack length $a+\Delta a$ under the same loading condition. Thus the defined energy release rate corresponds to the released energy when a crack grows suddenly from a to $a+\Delta a$. It is found that under a given loading history, the ERR is a function of time, and its maximum value should correspond with the critical state for delamination to propagate. Base on such definition of ERR, its analytical expression can be derived easily by getting rid of the effect of crack growth speed. Following William's work[14], the explicit expressions of ERR for widely used experimental configurations to measure the interfacial fracture toughness have been obtained through the classical beam assumption. Thus it becomes easier for us to use the standard experimental setups to measure the critical value of ERR for viscoelastic laminates.

1 General Formulations

It is a well-known fact that during deformation a viscoelastic material can store a certain part of energy. At the same time it will also dissipate a part of energy under a given loading history. Only the part of energy stored in the material contributes to the released energy during crack propagation. In this section of the paper, the stored energy and dissipated energy will be expressed in simple forms based on given constitutive relation.

For simplicity, here and in the sequel, we discuss only the case of small strain and displacements. The classical linear viscoelastic constitutive relation without aging can be written in from of Stieltjes integrals or Stieltjes convolutions as follows[4, 15]:

$$\sigma_{ij}(t) = \int_{0^-}^{t} r_{ijkl}(t-\tau)\mathrm{d}\varepsilon_{kl}(\tau), \tag{1}$$

$$\varepsilon_{ij}(t) = \int_{0^-}^{t} f_{ijkl}(t-\tau)\mathrm{d}\sigma_{kl}(\tau), \tag{2}$$

where the origin 0^- of time is taken previous to any loading, the kernels $r_{ijkl}(t)$ (relaxation function) and $f_{ijkl}(t)$ (creep function) are tensors of the fourth rank. They are suppose to exhibit the classical symmetric relations encountered in linear elasticity. These properties of symmetry come partly from the symmetry of the strain and stress tensor ε_{ij}, σ_{ij}, and partly from some specific assumption like applicability of Onsager principle in the problem. Eqs. (1) and (2) include the case where the strain history or stress history has a step jump at $t = 0$. If we take 0 instead of 0^-, Eqs. (1) and (2) give the constitutive relations for continuous strain or stress history.

For the given constitutive law, the elastic energy stored per unit volume at time t can be expressed as follows[15]:

$$W(t) = \frac{1}{2}\int_{0^-}^{t}\int_{0^-}^{t} r_{ijkl}(2t-u-v)\mathrm{d}\varepsilon_{ij}(u)\mathrm{d}\varepsilon_{kl}(v) = \sigma_{ij}(t)\varepsilon_{ij}(t) - \frac{1}{2}\int_{0^-}^{t}\int_{0^-}^{t} f_{ijkl}(2t-u-v)\mathrm{d}\sigma_{ij}(u)\mathrm{d}\sigma_{kl}(v). \tag{3}$$

The rate of the dissipated energy density can be written as

$$D(t) = -\int_{r}^{t} 0^- \int_{0^-}^{t} \dot{r}_{ijkl}(2t-u-v)\mathrm{d}\varepsilon_{ij}(u)\mathrm{d}\varepsilon_{kl}(v) = \int^{t} \dot{f}_{0^-} \int_{0^-}^{t} \dot{f}_{ijkl}(2t-u-v)\mathrm{d}\sigma_{ij}(u)\mathrm{d}\sigma_{kl}(v) \geqslant 0, \tag{4}$$

where $\dot{r}_{ijkl}(2t-u-v)$, $\dot{f}_{ijkl}(2t-u-v)$ means a derivative with respect to time t. As a result, if we can derive the strain or stress field for a given loading history, the stored energy and dissipated energy can be obtained through Eqs. (3) and (4).

Following the work of William's[14] on elastic laminate, we consider a delamination as shown in Fig. 1, where the upper and lower layers are thin viscoelastic sheets with width B and thickness h, respectively. The loading will be considered to be uniform in the width direction giving uniform conditions along the crack front. Let us consider the end of the delamination in which bending moment $M_1(t)$, $M_2(t)$ are applied to the upper and lower sections respectively at the section AB. By assuming that the crack grows from AB to CD instantaneously at some load, the energy release rate can be expressed in the form as

$$G = \frac{1}{B}\left(\frac{\mathrm{d}U_e}{\mathrm{d}a} - \frac{\mathrm{d}U_s}{\mathrm{d}a}\right), \tag{5}$$

where U_s is the stored energy in the specimen, and U_e is the external work performed.

For the case of a linearly elastic beam under pure bending, the equations for strain, stress and deflection are as follows:

$$\varepsilon(y) = -\frac{My}{EI}, \tag{6}$$

$$\sigma(y) = -\frac{My}{I}, \tag{7}$$

$$\frac{\mathrm{d}^2 w}{\mathrm{d}x^2} = \frac{\mathrm{d}\phi}{\mathrm{d}x} = \frac{M}{EI}, \tag{8}$$

where E is the axial Young's modulus, ϕ is the rotation of the cross-section, and I is the second moment of the area. For a rectangular beam, I is given by

$$I = \int_{-h}^{h} BH^2\mathrm{d}H = \frac{2}{3}Bh^3, \tag{9}$$

where B and $2h$ are width and height of the rectangular cross-section.

In the following, we use the elastic-viscoelastic correspondence principle to derive the counterparts of Eqs. (6), (7) and (8) in viscoelastic casein order to obtain the energy release rate as shown in Eq. (5):

1) Replacing E, the elastic modulus, by $s\bar{E}(s)$, the Laplace transformed quantity, one obtains the corresponding Laplace transformed quantities as follows:

$$\bar{\varepsilon}(y,s) = -\frac{y}{I}\cdot\frac{M}{s\bar{E}(s)}, \quad \frac{\mathrm{d}\bar{\phi}}{\mathrm{d}x} = \frac{1}{I}\cdot\frac{\bar{M}}{s\bar{E}(s)}; \tag{10}$$

2) Taking the inverse Laplace transformation of Eq. (10), the strain and rotation for a viscoelastic beam are obtained as follows:

$$\varepsilon(y,t) = -\frac{y}{I}\int_{-\infty}^{t} f(t-\tau)\frac{\mathrm{d}M(\tau)}{\mathrm{d}\tau}\mathrm{d}\tau, \quad \frac{\mathrm{d}\phi}{\mathrm{d}x} = \frac{1}{I}\int_{-\infty}^{t} f(t-\tau)\frac{\mathrm{d}M(\tau)}{\mathrm{d}\tau}\mathrm{d}\tau, \tag{11}$$

where $f(t-\tau)$ is the creep function as defined before.

If the crack tip is taken to be originally at point O on cross-section AB and then to move to point O' on CD (Fig. 1), we may take the original rotation as ϕ_0 at CD and $\phi_0 + (\mathrm{d}\phi_0/\mathrm{d}a)\delta a$ at AB. When the crack moves from O to O', the change in angle in the upper and lower beams respectively at cross-section AB is

$$\left(\frac{\mathrm{d}\phi_1}{\mathrm{d}a} - \frac{\mathrm{d}\phi_0}{\mathrm{d}a}\right)\delta a, \quad \left(\frac{\mathrm{d}\phi_2}{\mathrm{d}a} - \frac{\mathrm{d}\phi_0}{\mathrm{d}a}\right)\delta a, \tag{12}$$

respectively.

The work done by the applied load when the crack propagates from points O to O' is

$$\delta U_e = M_1(t)\left(\frac{\mathrm{d}\phi_1}{\mathrm{d}a} - \frac{\mathrm{d}\phi_0}{\mathrm{d}a}\right)\delta a + M_2(t)\left(\frac{\mathrm{d}\phi_2}{\mathrm{d}a} - \frac{\mathrm{d}\phi_0}{\mathrm{d}a}\right)\delta a. \tag{13}$$

Therefore we have

$$\begin{aligned}\frac{\mathrm{d}U_e}{\mathrm{d}a} &= M_1(t)\left(\frac{\mathrm{d}\phi_1}{\mathrm{d}a} - \frac{\mathrm{d}\phi_0}{\mathrm{d}a}\right) + M_2(t)\left(\frac{\mathrm{d}\phi_2}{\mathrm{d}a} - \frac{\mathrm{d}\phi_0}{\mathrm{d}a}\right) = \\ &M_1(t)\left[\frac{1}{I_1}\int_{-\infty}^{t} f_1(t-\tau)\frac{\mathrm{d}M_1(\tau)}{\mathrm{d}\tau}\mathrm{d}\tau - \frac{1}{I}\int_{-\infty}^{t} f(t-\tau)\frac{\mathrm{d}M(\tau)}{\mathrm{d}\tau}\mathrm{d}\tau\right] + \\ &M_2(t)\left[\frac{1}{I_2}\int_{-\infty}^{t} f_2(t-\tau)\frac{\mathrm{d}M_2(\tau)}{\mathrm{d}\tau}\mathrm{d}\tau - \frac{1}{I}\int_{-\infty}^{t} f(t-\tau)\frac{\mathrm{d}M(\tau)}{\mathrm{d}\tau}\mathrm{d}\tau\right],\end{aligned} \tag{14}$$

where the lower index 1 and 2 means the quantities corresponding to the lower and upper beams, and I, M and f are the quantities of the composite. The change of the stored elastic energy for the crack to propagate δa is the difference between the stored energy of two beams with length δa under the action of $M_1(t)$, $M_2(t)$, respectively, and the composite beam with the same length under the action $M(t)$, here

$$M(t) = M_1(t) + M_2(t). \tag{15}$$

The stored energy can thus be calculated as

$$\begin{aligned}\frac{\mathrm{d}U_s}{\mathrm{d}a} = &-\frac{B}{2I}\int_{-h/2}^{h/2}\mathrm{d}y\left[\int_0^t\int_0^t r(2t-u-v)\frac{\mathrm{d}\varepsilon(u)}{\mathrm{d}u}\frac{\mathrm{d}\varepsilon(v)}{\mathrm{d}v}\mathrm{d}u\mathrm{d}v\right] + \\ &\frac{B}{2I_1}\int_{-h_1/2}^{h_1/2}\mathrm{d}y\left[\int_0^t\int_0^t r_1(2t-u-v)\frac{\mathrm{d}\varepsilon_1(u)}{\mathrm{d}u}\frac{\mathrm{d}\varepsilon_1(v)}{\mathrm{d}v}\mathrm{d}u\mathrm{d}v\right] + \\ &\frac{B}{2I_2}\int_{-h_2/2}^{h_2/2}\mathrm{d}y\left[\int_0^t\int_0^t r_2(2t-u-v)\frac{\mathrm{d}\varepsilon_2(u)}{\mathrm{d}u}\frac{\mathrm{d}\varepsilon_2(v)}{\mathrm{d}v}\mathrm{d}u\mathrm{d}v\right],\end{aligned} \tag{16}$$

where $r_1(t-\tau)$, $r_2(t-\tau)$, $r(t-\tau)$ are relaxation functions of the lower, upper and composite beams, $\varepsilon_1(t)$, $\varepsilon_2(t)$, $\varepsilon(t)$ are corresponding strains under the action of $M_1(t)$, $M_2(t)$, $M(t)$, and in deriving Eq. (16), we assume that the crack propagates from O to O' instantaneously. Substitution of Eqs. (14) and (16) into Eq. (5) gives the energy release rate for the interlaminar crack to propagate in a viscoelastic composite laminate. In what follows, we consider the two loading histories.

1) $M(t) = M_0\mathrm{H}(t)$, $M_1(t) = M_{01}\mathrm{H}(t)$, $M_2(t) = M_{02}\mathrm{H}(t)$,

where H(t) is the Heaviside function. For such loading case, the strain and rotation can be expressed in the form as

$$\begin{cases} \varepsilon(t) = -\frac{y}{I}M_0 f(t), \quad \varepsilon_1(t) = -\frac{y}{I_1}M_{01}f_1(t), \quad \varepsilon_2(t) = -\frac{y}{I_2}M_{02}f_2(t), \\ \frac{\mathrm{d}\phi}{\mathrm{d}a} = \frac{1}{I}M_0 f(t), \quad \frac{\mathrm{d}\phi_1}{\mathrm{d}a} = \frac{1}{I_1}M_{01}f_1(t), \quad \frac{\mathrm{d}\phi_2}{\mathrm{d}a} = \frac{1}{I_2}M_{02}f_2(t). \end{cases} \tag{17}$$

Substitution of Eq. (17) into Eqs. (14) and (16), then into Eq. (5) yields

$$\begin{aligned} G = {} & \frac{M_{01}\mathrm{H}(t)}{B}\left[\frac{1}{I_1}M_{01}f_1 - \frac{1}{I}M_0 f\right] + \frac{M_{02}\mathrm{H}(t)}{B}\left[\frac{1}{I_2}M_{02}f_2 - \frac{1}{I}M_0 f\right] + \\ & \frac{M_0^2}{2BI}\int_0^t\int_0^t r(2t - u - v)f'(u)f'(v)\mathrm{d}u\mathrm{d}v - \\ & \frac{M_{01}^2}{2BI_1}\int_0^t\int_0^t r_1(2t - u - v)f_1'(u)f_1'(v)\mathrm{d}u\mathrm{d}v - \\ & \frac{M_{02}^2}{2BI_2}\int_0^t\int_0^t r_2(2t - u - v)f_2'(u)f_2'(v)\mathrm{d}u\mathrm{d}v. \end{aligned} \tag{18}$$

According to Eq. (18), if we know the constitutive relation of the beams and the applied moments at the crack tip, the energy release rate can be calculated easily.

2) $M(t) = M_0 t, \quad M_1 = M_{01} t, \quad M_2 = M_{02} t.$

For such linearly increasing load, the strain and rotation of a beam are given as follows:

$$\varepsilon = -\frac{M_0 y}{I}\int_0^t f(t - \tau)\mathrm{d}\tau = -\frac{M_0 y}{I}J(t), \tag{19}$$

$$\frac{\mathrm{d}\phi}{\mathrm{d}a} = \frac{M_0}{I}\int_0^t f(t - \tau)\mathrm{d}\tau = \frac{M_0}{I}J(t). \tag{20}$$

Therefore the energy release rate is given by

$$\begin{aligned} G = {} & \frac{M_{01}t}{B}\left[\frac{1}{I_1}M_{01}J_1(t) - \frac{1}{I}M_0 J(t)\right] + \frac{M_{02}t}{B}\left[\frac{1}{I_2}M_{02}J_2(t) - \frac{1}{I}M_0 J(t)\right] + \\ & \frac{M_0^2}{2BI}\int_0^t\int_0^t r(2t - u - v)J'(u)J'(v)\mathrm{d}u\mathrm{d}v - \\ & \frac{M_{01}^2}{2BI_1}\int_0^t\int_0^t r_1(2t - u - v)J_1'(u)J_2'(v)\mathrm{d}u\mathrm{d}v - \\ & \frac{M_{02}^2}{2BI_2}\int_0^t\int_0^t r_2(2t - u - v)J_2'(u)J_2'(v)\mathrm{d}u\mathrm{d}v. \end{aligned} \tag{21}$$

Equations (18) and (21) can be used to calculate the energy release rate for viscoelastic laminate from the local moments at the crack tip. whatever happens in the rest of the beam is uniportant. In what follows, we will use the configuration of *DCB* test as an example to derive the energy release rate for particular cases.

2 Examples

The most common test for mode Ⅰ is shown in Fig. 2, where we assume that the upper beam is viscoelastic, and the lower beam pure elastic. For symmetrical loading, we have $M_2 = -M_1 = pa$. The creep function of the upper viscoelastic beam is taken as

$$f_1(t) = f_{01}(1 - \mathrm{e}^{-t/t_1})\mathrm{H}(t). \tag{22}$$

Since

$$\int_0^t f(t-\tau)r(\tau)\mathrm{d}\tau = \int_0^t r(t-\tau)f(\tau)\mathrm{d}\tau = t, \tag{23}$$

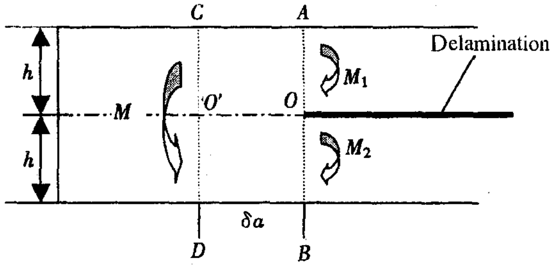

Fig.1 Delamination geometry in composite laminate

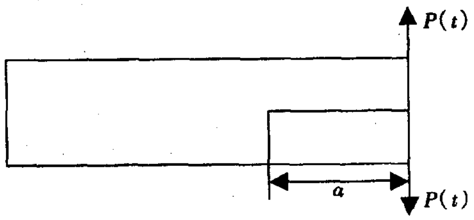

Fig.2 The double cantilever beam (DCB) testing configuration

one obtains

$$r_1(t) = \frac{1}{f_{01}} + \frac{t_1}{f_{01}}\delta(t). \tag{24}$$

The lower beam is elastic material, therefore

$$r_2 = E_2,\ f_2 = 1/E_2. \tag{25}$$

According to the rule of mixture, one can derive the relaxation function of the composite as follows:

$$r = \frac{1}{2}r_1 + \frac{1}{2}r_2 = \frac{1}{2}\left(E_2 + \frac{1}{f_1}\right) + \frac{1}{2}\frac{t_1}{f_{01}}\delta(t), \tag{26}$$

$$f(t) = f_0(1 - \mathrm{e}^{-t/t'})\mathrm{H}(t), \tag{27}$$

where

$$f_0 = \frac{2f_{01}}{1 + E_2 f_{01}},\quad t' = \frac{t_1}{1 + E_2 f_{01}}. \tag{28}$$

For such a laminate, Eq.(18) becomes

$$G = \frac{M_{01}\mathrm{H}(t)}{B}\left[\frac{1}{I_1}M_{01}f_1 - \frac{1}{I}M_0 f\right] + \frac{M_{02}\mathrm{H}(t)}{B}\left[\frac{1}{I_2}M_{02}f_2 - \frac{1}{I}M_0 f\right] + \frac{M_0^2}{2BI}\int_0^t\int_0^t r(2t-u-v)f'(u)f'(v)\mathrm{d}u\mathrm{d}v - \frac{M_{01}^2}{2BI_1}\int_0^t\int_0^t r_1(2t-u-v)f_1'(u)f_1'(v)\mathrm{d}u\mathrm{d}v - \frac{M_{02}^2}{2BE_2 I_2}. \tag{29}$$

Equation (21) becomes

$$G = \frac{M_{01}t}{B}\left[\frac{1}{I_1}M_{01}J_1(t) - \frac{1}{I}M_0 J(t)\right] + \frac{M_{02}t}{B}\left[\frac{1}{I_2}M_{02}J_2(t) - \frac{1}{I}M_0 J(t)\right] + \frac{M_0^2}{2BI}\int_0^t\int_0^t r(2t-u-v)J'(u)J'(v)\mathrm{d}u\mathrm{d}v - \frac{M_{01}^2}{2BI_1}\int_0^t\int_0^t r_1(2t-u-v)J_1'(u)J_1'(v)\mathrm{d}u\mathrm{d}v - \frac{(M_{02}t)^2}{2BI_2 I_2}. \tag{30}$$

For the loading condition $P = p_0 \mathrm{H}(t)$, substituting the constitutive relations (22) - (27) into Eq.(29), one obtains the energy release rate as follows:

$$G = \frac{p_0^2 a^2}{BI_1}\left\{f_{01}\left[1 - \mathrm{e}^{-t/t_1} - \frac{1}{2}(1 - \mathrm{e}^{-t/t_1})^2 - \frac{1}{2}\frac{t}{t_1}\mathrm{e}^{-2t/t_1}\right] + \frac{1}{2}\frac{1}{E_2}\right\}, \tag{31}$$

where $I_1 = Bh^3/12$. For electronic packages, the laminate can be considered to consist of two layers, one is the die, the other layer is epoxy molding compound, the material constants are as follows:

$$E_2 = 150\,\mathrm{GPa}, \quad f_{01} = 0.1(1/\mathrm{GPa}), \quad t_1 = 10^{12}(s). \tag{32}$$

The dimensionless energy release rate G/G_0 versus time t/t_1 is shown in Fig.3, where $G_0 = p_0^2 a^2/(2BE_2 I_1)$.

For linearly increasing loading $P = p_0 t$, one can obtain

$$\left.\begin{aligned} J_1(t) &= \int_0^t f_1(t - \tau)\mathrm{d}\tau \doteq f_{01}(t - t_1) + f_{01}t_1\mathrm{e}^{-t/t_1}, \\ J(t) &= \int_0^t f(t - \tau)\mathrm{d}\tau = f_0(t - t') + f_0 t'\mathrm{e}^{-t/t'}. \end{aligned}\right\} \tag{33}$$

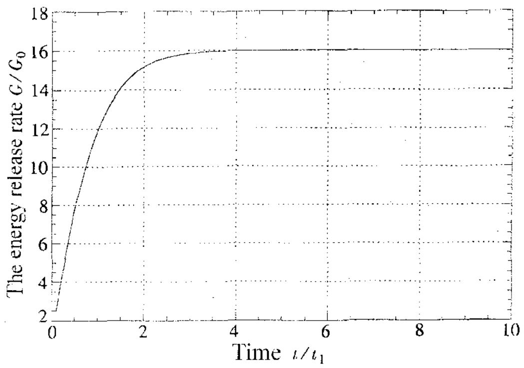

Fig.3 The energy release rate versus time for instantaneously applied loading

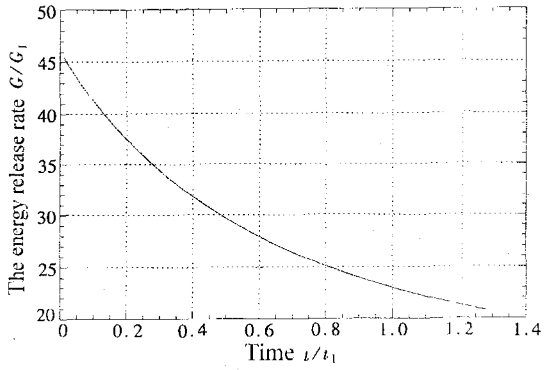

Fig.4 The energy release rate versus time for linearly increasing loading

Substitution of Eq.(33) into Eq.(30) yields

$$G = \frac{(M_{01}t)^2}{2BE_2 I_1}\left\{1 + E_2 f_{01}\left(1 - \frac{t_1}{t} + \frac{t_1}{t}\mathrm{e}^{-t/t_1}\right) - E_2 f_{01}\left(1 - \frac{t_1}{t} + \frac{t_1}{t}\mathrm{e}^{-t/t_1}\right)^2 + E_2 f_{01}\frac{t_1}{t}\left[1 - \frac{t_1}{t} + 2\mathrm{e}^{-t/t_1} + \left(2 + \frac{t_1}{t} + \frac{2}{3}\frac{t^2}{t_1^2}\right)\mathrm{e}^{-2t/t_1}\right]\right\}. \tag{34}$$

The energy release rate G/G_1, where $G_1 = (M_{01}t)^2/2BE_2 I_1$, versus time t/t_1 is shown in Fig.4. We can easily find that even if the generalized Maxwell constitutive relation for the viscoelastic layer is taken, the explicit expression for the energy release rate based on our definition can still be derived. The energy release rate is a function of time as expected for viscoelastic materials. At time t, it corresponds to the released energy per unit propagation length of the crack. The main assumption is that the crack propagates from a to $a + \Delta a$ instantaneously.

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