ACTA MECHANICA SOLIDA SINICA, Vol.16, No.1, March, 2003
Published by HUST, Wuhan, China

ISSN 0894-9166

ANALYSIS OF THE DYNAMIC BEHAVIOR OF A GRIFFITH PERMEABLE CRACK IN PIEZOELECTRIC MATERIALS WITH THE NON-LOCAL THEORY*

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Received 26 March 2002; revision received 8 October 2002

ABSTRACT The dynamic behavior of a Griffith permeable crack under harmonic anti-plane shear waves in the piezoelectric materials is investigated by use of the non-local theory. To overcome the mathematical difficulties, a one-dimensional non-local kernel is used instead of a two-dimensional one for the anti-plane dynamic problem to obtain the stress and the electric displacement near the crack tips. By means of Fourier transform, the problem can be solved with a pair of dual integral equations that the unknown variable is the jump of the displacement across the crack surfaces. These equations are solved with the Schmidt method and numerical examples are provided. Contrary to the previous results, it is found that no stress and electric displacement singularities are present at the crack tip. The finite hoop stress and the electric displacement depend on the crack length, the lattice parameter of the materials and the circle frequency of the incident waves. This enables us to employ the maximum stress hypothesis to deal with fracture problems in a natural way.

KEY WORDS crack, piezoelectric materials, non-local theory, Schmidt method

I. INTRODUCTION

In theoretical studies of crack problems in piezoelectric materials, several different electric boundary conditions at the crack surfaces have been proposed by numerous researchers[1]-[7]. For the sake of analytical simplification, the assumption that the crack surfaces are impermeable to electric fields was adopted in Refs.[1]-[2] and [4]-[7]. The impermeable crack assumption refers to the fact that the crack surfaces are free of surface charge and thus the electric displacement vanishes inside the crack. In fact, cracks in piezoelectric materials consist of vacuum, air or some other gases. This requires that the electric fields propagate through the crack, so the electric displacement component perpendicular to the crack surfaces should be continuous across the crack surfaces as stated in Refs.[2,3,5], where they analyzed the effects of electric boundary conditions at the crack surfaces on the fracture mechanics of piezoelectric materials. It is interesting to note that very different results were obtained by changing the boundary conditions[2]. The interaction of piezoelectric harmonic waves with impermeable cracks was analyzed in Refs.[4,6]. The semi-infinite propagating crack in a piezoelectric material with electrode boundary condition and vacuum condition on the crack surface respectively was studied in ref.[5]. However, these

* Project supported by the National Natural Science Foundation of China(Nos.10172030 and 50232030), the Natural Science Foundation of Heilongjiang Province, the Doctoral Science Foundation of Heilongjiang Province and the Multi-disciplinary Scientific Research Foundation of Harbin Institute of Techology(HT.MD.2001.39).

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solutions[1]-[7] contain the stress and electric displacement singularities. This is irrational from the physical point of view.

As is commonly known, one of the principal postulates of the traditional mechanics of continuous media is the principle of the local action. This principle excludes the action at a distance, and attributes changes occurring at a point of the medium to thermoenergetic factors acting at the point. Of necessity then, the classical theory, by restricting the response of the continuum to strictly local actions, constitutes a so-called local theory. A familiar example is provided by the conventional theory of elasticity, in which, when determining the stress at a point, one disregards the deformation and the temperature fields outside an arbitrarily small neighborhood at the point. However, the application of classical elasticity to micro-mechanics will lead to physically irrational results. A singularity appearing in a stress field is a typical one; the existence of stress singularities also results in difficulties in the development of experiments in micro-mechanics. In fact, the stress at the crack tip is finite. For this reason, beginning with Griffith, all fracture criteria in practice today are based on other considerations, e.g. energy, and the J-integral. In contrast to this local approach, of zero-range internal interactions, the modern non-local continuum mechanics, which originated and developed in the last four decades, postulates that the local state at a point is influenced by the action of all particles of the body. This was done primarily by Edelen[8]. According to the non-local theory, the stress at a point X in a body depends not only on the strain at point X but also on that at all other points of the body. This is different from the classical theory. According to the classical theory, the stress at a point X in a body depends only on the strain at point X. In Ref.[9], the basic theory of the non-local elasticity was stated with emphasis on the difference between the non-local theory and classical continuum mechanics. The basic idea of non-local elasticity is to establish a relationship between macroscopic mechanical quantities and microscopic physical quantities within the framework of continuum mechanics. The constitutive theory of non-local elasticity has been extensively developed[8], where the microstructures of the material have effect on the elastic modulus.

To overcome the stress singularity in the classical elastic theory, the state of stress near the tip of a sharp line crack in an elastic plane subjected to a uniform tension and anti-plane shear were discussed in Refs.[10,11] using the non-local theory. These solutions[10,11] gave finite stresses at the crack tip. For the past four decades, a number of non-local theories of mechanics has been advanced. Recently, the state of the dynamic stress near the tip of a line crack in an elastic strip has been investigated with the non-local theory in Ref.[12]. The static behavior of a crack in piezoelectric materials was studied with the non-local theory in Ref.[13]. These solutions[12,13] did not involve any stress and electric displacement singularities, thus having resolved the difficult fundamental problem that has remained unresolved over years. This enables us to employ the maximum stress hypothesis to deal with fracture problems in a natural way. To our knowledge, no work on the scattering of harmonic elastic anti-plane shear waves by a permeable crack in the piezoelectric materials has ever been reported.

In the present paper, the scattering of harmonic elastic anti-plane shear waves by a permeable crack in piezoelectric materials is investigated with the non-local theory. The traditional concept of linear elastic fracture mechanics and the non-local theory are extended to include the piezoelectric effects. To overcome mathematical difficulties, one has to accept some assumptions as given in Refs.[12,14], e.g. a one-dimensional non-local kernel function is used instead of a two-dimensional kernel function for the anti-plane dynamic problem to obtain the stress and electric displacement at the crack tips. Certainly, the assumption should be further investigated to satisfy the actual condition. Fourier transform technology is applied and a mixed boundary value problem is reduced to a pair of dual integral equations. In solving the dual integral equations, the jump of the displacement across the crack surface is expanded in a series of Jacobi polynomials. This process is quite different from those adopted in previous investigations as mentioned above Refs.[1]-[6] and [10,11]. As expected, the solution in this paper does not involve the stress and electric displacement singularities at the crack tip.

II. BASIC EQUATIONS OF NON-LOCAL PIEZOELECTRIC MATERIALS

For the anti-plane shear problem, the basic equations of linear, homogeneous, isotropic, non-local piezoelectric materials with vanishing body force, are[12,13]

$$\frac{\partial \tau_{xz}}{\partial x}+\frac{\partial \tau_{yz}}{\partial y}=\rho\frac{\partial^2 w}{\partial t^2} \tag{1}$$

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$$\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} = 0 \tag{2}$$

$$\tau_{kz}(X,t) = \int_V [c'_{44}(|X'-X|)\, w_{,k}(X',t) + e'_{15}(|X'-X|)\, \phi_{,k}(X',t)]\mathrm{d}V(X') \tag{3}$$

$$D_k(X,t) = \int_V [e'_{15}(|X'-X|) w_{,k}(X',t) - \varepsilon'_{11}(|X'-X|)\phi_{,k}(X',t)]\mathrm{d}V(X') \tag{4}$$

where the only difference from the classical electro-elastic theory is that the stress $\tau_{zk}(X,t)$ and the electric displacement $D_k(X,t)$ at a point X depends on $w_{,k}(X,t)$ and $\phi_{,k}(X,t)$, at all points of the body, where w and ϕ are the mechanical displacement and electric potential. For homogeneous and isotropic piezoelectric materials there exist only three material parameters, $c'_{44}(|X'-X|)$, $e'_{15}(|X'-X|)$ and $\varepsilon'_{11}(|X'-X|)$, which are functions of the distance $|X'-X|$. ρ is the density of the piezoelectric materials. The integrals in Eqs.(3)and (4) are over the volume V of the body enclosed within a surface ∂V.

As discussed in Ref.[13], the form of $c'_{44}(|X'-X|)$, $e'_{15}(|X'-X|)$ and $\varepsilon'_{11}(|X'-X|)$ can be expressed as

$$(c'_{44}, e'_{15}, \varepsilon'_{11}) = (c_{44}, e_{15}, \varepsilon_{11})\alpha(|X'-X|) \tag{5}$$

where $\alpha(|X'-X|)$ is known as the influence function and is a function of the distance $|X'-X|$, c_{44}, e_{15} and ε_{11} are the shear modulus, piezoelectric coefficient and dielectric parameter, respectively.

Substitution of Eq.(5) into Eqs.(3) and (4) yields

$$\tau_{kz}(X,t) = \int_V \alpha(|X'-X|)\sigma_{kz}(X',t)\mathrm{d}V(X') \qquad (k=x,y) \tag{6}$$

$$D_k(X,t) = \int_V \alpha(|X'-X|)D_k^{\mathrm{c}}(X',t)\mathrm{d}V(X') \qquad (k=x,y) \tag{7}$$

where

$$\sigma_{kz} = c_{44}w_{,k} + e_{15}\phi_{,k} \tag{8}$$

$$D_k^{\mathrm{c}} = e_{15}w_{,k} - \varepsilon_{11}\phi_{,k} \tag{9}$$

The expressions (8,9) are the classical constitutive equations.

III. THE CRACK MODEL

Consider an infinite piezoelectric body containing a Griffith permeable crack of length $2l$ along the x-axis. The geometry of the problem is shown in Fig.1. In this paper, the harmonic anti-plane wave is vertically incident. Let ω be the circular frequency of the incident wave. $-\tau_0$ is the magnitude of the incident wave. In what follows, the time dependence of all field quantities assumed to be of the form $\mathrm{e}^{-\mathrm{i}\omega t}$ will be suppressed but understood. The piezoelectric boundary-value problem for anti-plane shear is considerably simplified if we consider only the out-of-plane displacement and the in-plane electric fields. Since no opening displacement exists for the present anti-plane problem, the crack surfaces can be assumed to be in perfect contact. Accordingly, the permeability condition will be enforced in the present study, i.e., both the electric potential and the normal electric displacement are assumed to be continuous across the crack surfaces. So the permeable boundary conditions on the crack faces at $y=0$ are

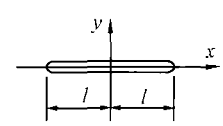

Fig. 1 Crack in the piezoelectric materials body.

$$\tau_{yz}^{(1)}(x,0^+,t) = \tau_{yz}^{(2)}(x,0^-,t) = -\tau_0 \qquad (|x| \le l) \tag{10}$$

$$D_y^{(1)}(x,0^+,t) = D_y^{(2)}(x,0^-,t), \quad \phi^{(1)}(x,0^+,t) = \phi^{(2)}(x,0^-,t) \qquad (|x| \le \infty) \tag{11}$$

$$w^{(1)}(x,0^+,t) = w^{(2)}(x,0^-,t) = 0 \qquad (|x| > l) \tag{12}$$

$$w^{(k)}(x,y,t) = \phi^{(k)}(x,y,t) = 0, \quad \text{for} \quad (x^2+y^2)^{1/2} \to \infty \qquad (k=1,2) \tag{13}$$

Note that all quantities with superscript k (k=1, 2) refer to the upper half plane and lower half plane.

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Substituting Eqs.(6)-(9) into Eqs.(1) and (2), respectively, and using Green-Gauss theorem, we have[13]

$$\iint_V \alpha(|x'-x|,|y'-y|)[c_{44}\nabla^2 w(x',y',t)+e_{15}\nabla^2\phi(x',y',t)]\mathrm{d}x'\mathrm{d}y' - \int_{-l}^{l}\alpha(|x'-x|,0)\underline{\sigma_{yz}(x',0,t)}\mathrm{d}x'=0 \tag{14}$$

$$\iint_V \alpha(|x'-x|,|y'-y|)[e_{15}\nabla^2 w(x',y',t)-\varepsilon_{11}\nabla^2\phi(x',y',t)]\mathrm{d}x'\mathrm{d}y' - \int_{-l}^{l}\alpha(|x'-x|,0)\underline{D_y^c(x',0,t)}\mathrm{d}x'=0 \tag{15}$$

where the underline indicates a jump at the crack line. i.e. $\underline{\sigma_{yz}(x',0,t)}=\sigma_{yz}(x',0^+,t)-\sigma_{yz}(x',0^-,t)$, and $\nabla^2=\partial^2/\partial x^2+\partial^2/\partial y^2$ is the two-dimensional Laplace operator. Because of the assumed symmetry in geometry and loading, it is sufficient to consider the problem for $0\le x\le\infty$, $0\le y\le\infty$ only. Under the applied anti-plane shear load on the unopened surfaces of the crack, the displacement field and the electric displacement possess the following symmetry regulations:

$$w(x,-y,t)=-w(x,y,t),\quad \phi(x,-y,t)=-\phi(x,y,t) \tag{16}$$

Using Eq.(16), we find that

$$\underline{\sigma_{yz}(x,0,t)}=0 \tag{17}$$

$$\underline{D_y^c(x,0,t)}=0 \tag{18}$$

Hence the line integrals in Eqs.(14) and (15) vanish. By taking the Fourier transform of Eqs.(14) and (15) with respect to x', it can be shown that the general solutions of Eqs.(14) and (15) are identical to that of

$$\int_0^\infty \bar{\alpha}(|s|,|y'-y|)\left\{c_{44}\left[\frac{\mathrm{d}^2\bar{w}(s,y',t)}{\mathrm{d}y^2}-s^2\bar{w}(s,y',t)\right] + e_{15}\left[\frac{\mathrm{d}^2\bar{\phi}(s,y',t)}{\mathrm{d}y^2}-s^2\bar{\phi}(s,y',t)\right]\right\}\mathrm{d}y'=-\rho\omega^2\bar{w} \tag{19}$$

$$\int_0^\infty \bar{\alpha}(|s|,|y'-y|)\left\{e_{15}\left[\frac{\mathrm{d}^2\bar{w}(s,y',t)}{\mathrm{d}y^2}-s^2\bar{w}(s,y',t)\right] - \varepsilon_{11}\left[\frac{\mathrm{d}^2\bar{\phi}(s,y',t)}{\mathrm{d}y^2}-s^2\bar{\phi}(s,y',t)\right]\right\}\mathrm{d}y'=0 \tag{20}$$

Here a superposed bar indicates the Fourier transform, e.g.

$$\bar{f}(s,y)=\int_0^\infty f(x,y)\exp(\mathrm{i}sx)\mathrm{d}x$$

What now remains is to solve the integrodifferential equations (19) and (20) for the functions w and ϕ. It seems obviously that a rigorous solution of such a problem encounters serious mathematical difficulties, and one has to resort to an approximate procedure. In this given problem, according to the assumptions as made in Refs.[12,14], the non-local interaction in y-direction will be ignored. It can be given as

$$\bar{\alpha}(|s|,|y-y|)=\bar{\alpha}_0(s)\delta(y-y) \tag{21}$$

From Eqs.(19) and (20), it can be shown that

$$\bar{\alpha}_0(s)\left\{c_{44}\left[\frac{\mathrm{d}^2\bar{w}(s,y,t)}{\mathrm{d}y^2}-s^2\bar{w}(s,y,t)\right] + e_{15}\left[\frac{\mathrm{d}^2\bar{\phi}(s,y,t)}{\mathrm{d}y^2}-s^2\bar{\phi}(s,y,t)\right]\right\}=-\rho\omega^2\bar{w} \tag{22}$$

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$$e_{15}\left[\frac{\mathrm{d}^2\bar{w}(s,y,t)}{\mathrm{d}y^2}-s^2\bar{w}(s,y,t)\right]-\varepsilon_{11}\left[\frac{\mathrm{d}^2\bar{\phi}(s,y,t)}{\mathrm{d}y^2}-s^2\bar{\phi}(s,y,t)\right]=0 \tag{23}$$

The general solutions of Eqs.(22) and (23) satisfying Eq.(13) are, respectively,

$$w^{(1)}(x,y,t)=\frac{2}{\pi}\int_0^\infty A_1(s)\mathrm{e}^{-\gamma y}\cos(xs)\mathrm{d}s \tag{24}$$

$$\phi^{(1)}(x,y,t)-\frac{e_{15}}{\varepsilon_{11}}w^{(1)}(x,y,t)=\frac{2}{\pi}\int_0^\infty B_1(s)\mathrm{e}^{-sy}\cos(xs)\mathrm{d}s \tag{25}$$

$$w^{(2)}(x,y,t)=\frac{2}{\pi}\int_0^\infty A_2(s)\mathrm{e}^{\gamma y}\cos(xs)\mathrm{d}s \tag{26}$$

$$\phi^{(2)}(x,y,t)-\frac{e_{15}}{\varepsilon_{11}}w^{(2)}(x,y,t)=\frac{2}{\pi}\int_0^\infty B_2(s)\mathrm{e}^{sy}\cos(xs)\mathrm{d}s \tag{27}$$

where $\gamma^2=s^2-\omega^2/[c^2\bar{\alpha}_0(s)]$, $c^2=\mu/\rho$, $\mu=c_{44}+e_{15}^2/\varepsilon_{11}$. $A_1(s)$, $B_1(s)$, $A_2(s)$ and $B_2(s)$ are to be determined from the boundary conditions.

The stress field and the electric displacement, according to Eqs.(6) and(7), are given by, respectively

$$\tau_{yz}^{(1)}(x,0^+,t)=-\frac{2}{\pi}\int_0^\infty \bar{\alpha}_0(s)[\mu\gamma A_1(s)+e_{15}sB_1(s)]\cos(sx)\mathrm{d}s \tag{28}$$

$$D_y^{(1)}(x,0^+,t)=\frac{2}{\pi}\int_0^\infty \varepsilon_{11}\bar{\alpha}_0(s)sB_1(s)\cos(sx)\mathrm{d}s \tag{29}$$

$$\tau_{yz}^{(2)}(x,0^-,t)=\frac{2}{\pi}\int_0^\infty \bar{\alpha}_0(s)[\mu\gamma A_2(s)+e_{15}sB_2(s)]\cos(sx)\mathrm{d}s \tag{30}$$

$$D_y^{(2)}(x,0^-,t)=-\frac{2}{\pi}\int_0^\infty \varepsilon_{11}\bar{\alpha}_0(s)sB_2(s)\cos(sx)\mathrm{d}s \tag{31}$$

To solve the problem, the jump functions of the displacements and the electric potentials across the crack surfaces are defined as follows:

$$f_w(x)=w^{(1)}(x,0^+,t)-w^{(2)}(x,0^-,t) \tag{32}$$

$$f_\phi(x)=\phi^{(1)}(x,0^+,t)-\phi^{(2)}(x,0^-,t) \tag{33}$$

Substituting Eqs.(24)-(27) into Eqs.(32) and (33), and applying the Fourier transform, we have

$$\bar{f}_w(s)=A_1(s)-A_2(s) \tag{34}$$

$$\bar{f}_\phi(s)=\frac{e_{15}}{\varepsilon_{11}}[A_1(s)-A_2(s)]+B_1(s)-B_2(s) \tag{35}$$

Substituting Eqs.(28)-(31) into Eqs.(10) and (11), we can obtain

$$-\mu\gamma[A_1(s)+A_2(s)]-e_{15}s[B_1(s)+B_2(s)]=0 \tag{36}$$

$$B_1(s)+B_2(s)=0 \tag{37}$$

$$\frac{e_{15}}{\varepsilon_{11}}[A_1(s)-A_2(s)]+B_1(s)-B_2(s)=0 \tag{38}$$

By solving four equations (34) and (36)-(38) with four unknown functions and applying the boundary conditions (10)-(13), we can obtain

$$\frac{1}{\pi}\int_0^\infty \bar{\alpha}_0(s)[\mu\gamma-\frac{e_{15}^2}{\varepsilon_{11}}s]\bar{f}_w(s)\cos(sx)\mathrm{d}s=\tau_0 \qquad (|x|\le l) \tag{39}$$

$$\frac{1}{\pi}\int_0^\infty \bar{f}_w(s)\cos(sx)\mathrm{d}s=0 \qquad (|x|>l) \tag{40}$$

To determine the unknown function $\bar{f}_w(s)$, the dual-integral equations (39) and (40) must be solved.

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IV. SOLUTION OF THE DUAL INTEGRAL EQUATIONS

The dual integral equations (39) and (40) cannot be transformed into Fredholm integral equations of the second kind or the first kind as discussed in Refs.[12,13]. This makes the numerical solution of such equations quite difficult. In this paper, the Schmidt method[12,13] was used to overcome the difficulty. As discussed in Refs.[10]-[13], we have

$$\alpha_0 = \chi_0 \exp\left[-(\beta/a)^2 (x' - x)^2\right] \tag{41}$$

$$\chi_0 = \beta/(a\sqrt{\pi}) \tag{42}$$

where β is a constant (here β is a constant appropriate to each material). a is the lattice parameter. So it can be found that

$$\bar{\alpha}_0(s) = \exp[-(sa)^2/(2\beta)^2] \tag{43}$$

and $\bar{\alpha}_0(s) = 1$ for the limit $a \to 0$, so that Eqs.(39) and (40) are reduced to a pair of dual integral equations for the same problem in the classical theory. Here the Schmidt method can be used to solve the dual integral equations (39) and (40). The jump functions of the displacement across the crack surfaces are represented by the following series:

$$f_w(x) = w^{(1)}(x,0^+,t) - w^{(2)}(x,0^-,t) = \sum_{n=1}^{\infty} a_n P_{2n-2}^{(\frac{1}{2},\frac{1}{2})}(\frac{x}{l})(1-\frac{x^2}{l^2})^{1/2} \quad \text{for } -l \le x \le l,\ y=0 \tag{44}$$

$$f_w(x) = w^{(1)}(x,0^+,t) - w^{(2)}(x,0^-,t) = 0 \quad \text{for } |x| > l,\ y=0 \tag{45}$$

where a_n represents unknown coefficients to be determined and $P_n^{(\frac{1}{2},\frac{1}{2})}(x)$ is a Jacobi polynomial. The Fourier transformation of Eqs.(44) and (45) is

$$\bar{f}_w(s) = \sum_{n=1}^{\infty} a_n G_n \frac{1}{s} J_{2n-1}(sl), \quad G_n = 2\sqrt{\pi}(-1)^{n-1}\frac{\Gamma(2n-1/2)}{(2n-2)!} \tag{46}$$

where $\Gamma(x)$ and $J_n(x)$ are the Gamma and Bessel functions, respectively.

By substituting Eq.(46) into Eqs.(39) and (40), respectively, Eq.(40) can be automatically satisfied. Then the remaining equation (39) is reduced to the form,

$$\sum_{n=1}^{\infty} a_n G_n \int_0^{\infty} \bar{\alpha}_0(s)\left[\frac{\mu\gamma}{s} - \frac{e_{15}^2}{\varepsilon_{11}}\right] J_{2n-1}(sl)\cos(sx)\mathrm{d}s = \pi\tau_0 \tag{47}$$

For a large s, the integrands of Eq.(47) almost decreases exponentially. So the semi-infinite integral in Eq.(47) can be evaluated numerically. Equation (47) can now be solved for the coefficients a_n by the Schmidt method[13].

V. NUMERICAL CALCULATIONS

From the literatures [12,13], it can be seen that the Schmidt method works satisfactorily if the first ten terms of infinite series to Eq.(47) are retained. The behavior of the stress stays steady with the increasing number of terms in Eq.(47). Coefficients a_n are known, so that the entire stress field and the electric displacement will be obtainable. However, in fracture mechanics, it is of importance to determine stress $\tau_{yz}^{(1)}$ and the electric displacement $D_y^{(1)}$ in the vicinity of the crack's tips. $\tau_{yz}^{(1)}$ and $D_y^{(1)}$ along the crack line can be expressed respectively as

$$\tau_{yz}^{(1)}(x,0,t) = -\frac{1}{\pi}\sum_{n=1}^{\infty} a_n G_n \int_0^{\infty} \bar{\alpha}_0(s)\left(\frac{\mu\gamma}{s} - \frac{e_{15}^2}{\varepsilon_{11}}\right) J_{2n-1}(sl)\cos(xs)\mathrm{d}s \tag{48}$$

$$D_y^{(1)}(x,0,t) = -\frac{e_{15}}{\pi}\sum_{n=1}^{\infty} a_n G_n \int_0^{\infty} \bar{\alpha}_0(s) J_{2n-1}(sl)\cos(xs)\mathrm{d}s \tag{49}$$

So long as $\varepsilon = a/(2\beta) \neq 0$, the semi-infinite integration and the series in the Eqs.(48) and (49) are convergent for any variable x. Equations (48) and (49) give finite stress and electric displacement all

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along $y = 0$, so there are no stress and electric displacement singularities at the crack tips. However, for $\varepsilon = 0$, we have the classical stress and electric displacement singularities at the crack tips. At $-l < x < l$, $\tau_{yz}^{(1)}/\tau_0$ is very close to unity, and for $x > l$, $\tau_{yz}^{(1)}/\tau_0$ possesses finite values diminishing from a finite value at $x = l$ to zero at $x = \infty$. Since $\varepsilon/l > 1/100$ represents a crack length of less than 100 atomic distances as stated in Ref.[11], such submicroscopic size problem and other serious questions may arise regarding the interatomic arrangements and force laws, so we do not pursue solutions valid at such small crack sizes. In no computation are the material constants considered because the stress field does not depend on them. There is just the lattice parameter that is considered in this paper.

VI. DISCUSSION

The aim of the present paper is to study the application of the non-local theory in fracture mechanics of the piezoelectric materials. The present paper is also intended to show that the Schmidt method can be used to solve this kind of dual integral equation and that the limit of the kernel does not tend to a constant. This method is more exact and more appropriate than that put forward by Eringen[11] for solving this kind of problem. Contrary to the classical-theory solution, it is found that no stress and electric displacement singularities are present at the crack tip and the stress is finite at the crack tip. Furthermore, the effects of the geometry of the interacting cracks, the frequency of the incident wave and the lattice parameter upon the dynamic stress and the electric displacement fields of the crack are examined. In this paper, we only made an attempt to relate our formulation to a problem in a lattice structure. However, there are many problems that should be investigated in future work on non-local theory. For example, the choice of the influence function α should be further studied to satisfy the actual conditions, the practical value of the maximum stress near the crack tips should be measured by experiment, and so on. The results are plotted in Figs.2 to 9. The following observations are of great significance:

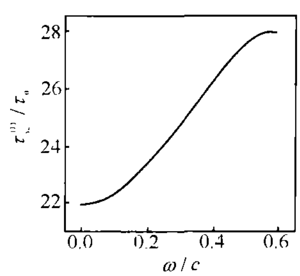

Fig. 2. The stress at the crack tip versus ω/c for $l = 1.0, a/(2\beta) = 0.0005$.

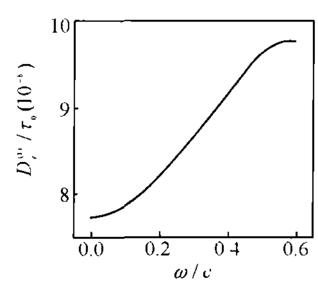

Fig. 3. The electric displacement at the crack tip versus ω/c for $l = 1.0, a/(2\beta) = 0.0005$.

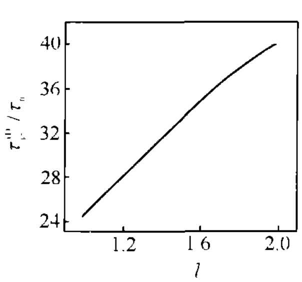

Fig. 4. The stress at the crack tip versus l for $\omega/c = 0.3, a/(2\beta) = 0.0005$.

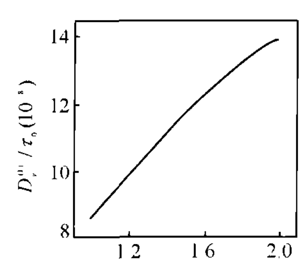

Fig. 5. The electric displacement at the crack tip versus l for $\omega/c = 0.3, a/(2\beta) = 0.0005$.

(i) For the lattice parameter $a \neq 0$, it can be proved that the semi-infinite integration and the series in Eqs.(48) and (49) are convergent for any variable x. So the stress and the electric displacement give

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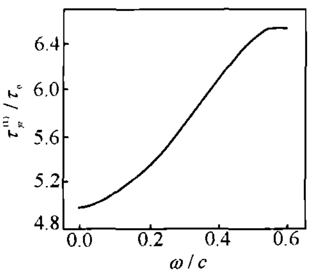

Fig. 6. The stress at the crack tip versus ω/c for $l = 1.0, a/(2\beta) = 0.008$.

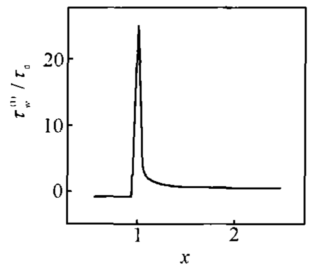

Fig. 7. The stress along the crack line versus x for $\omega/c = 0.3, l = 1.0, a/(2\beta) = 0.0005$.

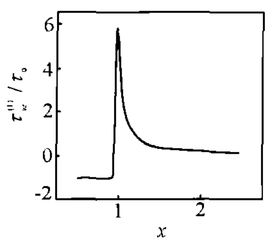

Fig. 8. The stress along the crack line versus x for $\omega/c = 0.3, l = 1.0, a/(2\beta) = 0.008$.

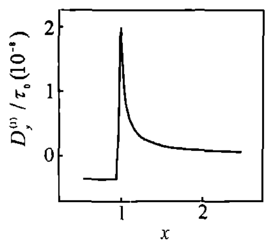

Fig. 9. The electric displacement along the crack line versus x for $\omega/c = 0.3, l = 1.0, a/(2\beta) = 0.008$.

finite values all along the crack line. Contrary to the classical piezoelectric theory solution, it is found that no stress and electric displacement singularities are present at the crack tip, and also the present results converge to the classical ones when far away from the crack tip. The maximum stress does not occur at the crack tip, but slightly away from it. This phenomenon has been thoroughly substantiated in Ref.[15]. The distance between the crack tip and the maximum stress point is very small, which depends on the crack length and the lattice parameter.

(ii) The stress at the crack tip becomes infinite as the lattice parameter $a \to 0$. This is the classical continuum limit of square root singularity. This can be shown from Eqs.(39) and (40). For the lattice parameter $a \to 0$, $\bar{\alpha}_0(s) = 1$, Eqs.(39) and (40) will reduce to the dual integral equations for the same problem in classical piezoelectric materials.

(iii) The value of the stress at the crack tip becomes higher with an increase of the crack length. It has been shown by experiment that piezoelectric materials with smaller cracks are more resistant to fracture than those with larger cracks.

(iv) In contrast to the impermeable crack surface condition solution as shown in Ref.[13], it is found that the perturbation electric displacement intensity factor for the permeable crack surface conditions is much smaller than the results for the impermeable crack surface conditions.

(v) The stress and the electric displacement become higher with an increase of the incident wave frequency. The anti-plane shear stress and the electric displacement at the crack tip increase with a decrease of the lattice parameter.

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