ACTA MECHANICA SOLIDA SINICA, Vol.13, No.4, December 2000 ISSN 0894 - 9166
Published by HUST, Wuhan, China

THE SCATTERING OF HARMONIC ELASTIC ANTI-PLANE SHEAR WAVES BY A FINITE CRACK IN INFINITELY LONG STRIP USING THE NON-LOCAL THEORY*

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ABSTRACT In this paper, the scattering of harmonic anti-plane shear waves by a finite crack in infinitely long strip is studied using the non-local theory. The Fourier transform is applied and a mixed boundary value problem is formulated. Then a set of dual integral equations is solved using the Schmidt method instead of the first or the second integral equation method. A one-dimensional non-local kernel is used instead of a two-dimensional one for the anti-plane dynamic problem to obtain the stress occurring at the crack tips. Contrary to the classical elasticity solution, it is found that no stress singularity is present at the crack tip. The non-local dynamic elastic solutions yield a finite hoop stress at the crack tip, thus allowing for a fracture criterion based on the maximum dynamic stress hypothesis. The finite hoop stress at the crack tip depends on the crack length, the width of the strip and the lattice parameter.

KEY WORDS non-local theory, Schmidt method, elastic wave, dual integral equation

Ⅰ. INTRODUCTION

The last four decades have witnessed the inauguration of a novel theory of material bodies, named non-local mechanics. This was made possible mainly because of the efforts of Eringen[1], Green and Rivlin[2], Kroener[3], and Kunin[4], and involved elastic, plastic (Eringen[5]) and liquid (Eringen[6]) media.

In several previous papers[7-9], Eringen discussed the state of stress near the tip of a sharp line crack in an elastic plate subject to uniform tension, shear and anti-plane shear. The field equations employed in the solution of these problems are those of the theory of non-local elasticity. These solutions gave finite stresses at the crack tips, thus resolving a fundamental problem that persisted over the years. This enables us to employ the maximum stress hypothesis to deal with fracture problems in a natural way, and the non-local elasticity makes it potentially possible to understand the behavior of composite materials. However, Eringen's[7-9] solution is not exact. The stress solution of Eringen's[7] has oscillations near the crack tip for one-dimensional problems. For a large lattice parameter, the relative errors of Eringen's[7] solution will become large. For this reason, the iterative technique used by Eringen[7] was not helpful to solving this kind of problem. The methods used by Eringen[8,9] were not helpful to solving dual-integral equation, either, because the kernel of the second kind of Fredholm in-

* Supported by the Post Doctoral Science Foundation of Heilongjiang Province, the Natural Science Foundation of Heilongjiang Province and the National Foundation for Excellent Young Investigators.
Received 12 July 2000.

tegral equation in Eringen's papers[8,9] is divergent. And in papers[10,11], they discussed the propagation of Love wave, the dispersion of plane waves and the wave propagation in elastic plate by use of non-local theory. In fracture mechanics, it is important to study the behavior of dynamic stress in the vicinity of the crack end, and the dynamic crack problems of the strip are of particular interest. In recent papers[12-14], the scattering of harmonic elastic waves by a finite crack or two collinear cracks using the non-local theory was investigated. To the author's knowledge, the dynamic crack problem[15] in the strip using the classical elastic theory has been tackled for a long time . However, no analytical treatment of the dynamic crack problem in the strip by using the non-local theory has ever been attempted.

The present paper deals with the problem of a line crack in an infinitely long strip where the crack surface is subjected to the harmonic anti-plane shear wave. The field equations of non-local elasticity theory are employed to formulate and solve this problem. For overcoming the mathematical difficulties, one-dimensional non-local kernel function is used instead of two-dimensional kernel function for the anti-plane dynamic problem to obtain the stress occurring at the crack tips. For obtaining the theoretical solution and discussing the probability of using the non-local theory to solve the dynamic fracture strip problem, one has to accept some assumptions made by Nowinski[10,11]. Certainly, the assumption should be further investigated to satisfy the realistic condition. In solving the equations, the crack surface displacement is expanded in a series of Jacobi polynomials and Schmidt method[16]. This process is quite different from that adopted in Refs. [7-9,15]. This method can overcome difficulties that occur in Eringen's papers[7-9]. The solution in this paper is more accurate and reasonable than Eringen's[7-9]. The solution of the present paper, as expected, does not contain the stress singularity near the crack tips, thus clearly indicating the physical nature of the problem, namely, in the vicinity of the geometrical discontinuities in the body, the non-local intermolecular forces are dominant. For such problems, therefore, one has to resort to theories incorporating non-local effects, at least in the neighbourhood of the discontinuities. The stress along the crack line depends on the crack length, the width of the strip and the lattice parameter.

Ⅱ. BASIC EQUATIONS OF NON-LOCAL ELASTICITY

According to the non-local theory, the stress at a point X in the body depends not only on the strains at X but also on strains at all other points of the body. This observation is in agreement with the atomic theory of lattice dynamics and experimental observations on phonon dispersion (Eringen[17]). In the limit when the effects of strains at points other than X are neglected, one obtains classical (local) theory of elasticity. For homogeneous and isotropic elastic solids, the linear theory is expressed by the set of equations with vanishing body force as follows:

$$\tau_{kl,k} = \rho \ddot{u}_l \tag{1}$$

$$\tau_{kl} = \int_V [\lambda'(|X' - X|) e_{rr}(X', t)\delta_{kl} + 2\mu'(|X' - X|) e_{kl}(X', t)] \mathrm{d}V(X') \tag{2}$$

$$e_{kl} = (u_{k,l} + u_{l,k})/2 \tag{3}$$

where the only difference from classical elasticity is in the stress constitutive Eq. (2) in which the stress $\tau_{kl}(X, t)$ at a point X depends on the strains $e_{kl}(X', t)$, at all points of the body. For homogeneous and isotropic solids there exist only two material constants, $\lambda'(|X' - X|)$ and $\mu'(|X' - X|)$ which are functions of the distance $|X' - X|$. The integral in Eq. (2) is over the volume V of the body enclosed within a surface ∂V. $\lambda'(|X' - X|)$ and $\mu'(|X' - X|)$ can be written as follows (Eringen et al.[7-9,18]):

$$(\lambda', \mu') = (\lambda, \mu)\alpha(|X' - X|) \tag{4}$$

$\alpha(|X' - X|)$ is known as influence function, and is the function of the distance $|X' - X|$. λ and μ are the Lamé constants of classical elasticity. ρ is the mass density of the material.

Substitution of Eq.(4) into Eq.(2) yields

$$\tau_{kl}(X, t) = \int_V \alpha(|X' - X|)\sigma_{kl}(X', t)\mathrm{d}V(X') \tag{5}$$

where

$$\begin{aligned}\sigma_{ij}(X', t) &= \lambda e_{rr}(X', t)\delta_{ij} + 2\mu e_{ij}(X', t) \\ &= \lambda u_{r,r}(X', t)\delta_{ij} + \mu[u_{i,j}(X', t) + u_{j,i}(X', t)]\end{aligned} \tag{6}$$

The expression of Eq.(6) is the classical Hooke's law. Substituting Eq.(6) into Eq.(1) and using Green-Gauss theorem, it can be shown that

$$\int_V \alpha(|X' - X|)[(\lambda + \mu)u_{k,kl}'(X', t) + \mu u_{l,kk}'(X', t)]\mathrm{d}V(X') - \int_{\partial V} \alpha(|X' - X|)\sigma_{kl}(X', t)\mathrm{d}a_k(X') = \rho\ddot{u}_l \tag{7}$$

Here the surface integral may be dropped if the only surface of the body is at infinity. $\mathrm{d}a_k$ is the element of the surface ∂V.

Ⅲ. THE CRACK MODEL

Consider an infinitely long, homogeneous isotropic thin elastic strip of width $2h$ containing a finite crack parallel to the edges of the strip. The crack occupies the region $|x| \leqslant l$, $y = 0$. The geometry of the problem is shown in Fig.1. Let ω be the circular frequency of the incident wave. In what follows, the time dependence of all field quantities assumed to be of the form $e^{-i\omega t}$ will be suppressed but understood. When the crack is subjected to the harmonic elastic anti-plane shear waves, as discussed in Ref.[19], the boundary conditions on the crack faces at $y = 0$ are

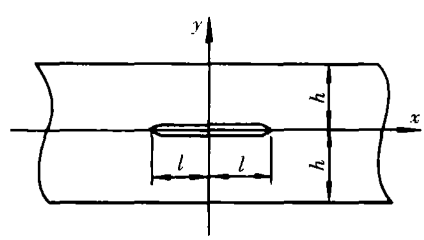

Fig.1 A finite elastic strip containing a central crack.

$$w(x, 0, t) = 0, \qquad |x| > l \tag{8}$$

$$\tau_{yz}(x, 0, t) = -\tau_0, \qquad |x| \leqslant l \tag{9}$$

$$\tau_{yz}(x, \pm h, t) = 0, \qquad -\infty < x < \infty \tag{10}$$

$$w(x, y, t) = 0, \qquad \text{for } x \to \pm\infty \tag{11}$$

$$\sigma_{xz} = \mu\frac{\partial w}{\partial x}, \ \sigma_{yz} = \mu\frac{\partial w}{\partial y}, \qquad \text{all other } \sigma_{kl} = 0 \tag{12}$$

In this paper, the wave is vertically incident and it is assumed that τ_0 is positive.

Ⅳ. THE DUAL INTEGRAL EQUATIONS

According to the boundary conditions, Eq.(7) can be written as follows:

$$\mu\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \alpha(|x' - x|, |y' - y|)\nabla^2 w(x', y', t)\mathrm{d}x'\mathrm{d}y' - \int_{-l}^{l} \alpha(|x' - x|, 0)[\sigma_{yz}(x', 0, t)]\mathrm{d}x' - \int_{-\infty}^{\infty} \alpha(|x' - x|, h)\{[\sigma_{yz}(x', h, t)] + [\sigma_{yz}(x', -h, t)]\}\mathrm{d}x' = -\rho\omega^2 w \tag{13}$$

where $$[\sigma_{yz}(x',y,t)] = \sigma_{yz}(x',y^+,t) - \sigma_{yz}(x',y^-,t) \tag{14}$$

The displacement field possesses the following symmetry regulation:

$$w(x,y,t) = -w(x,-y,t) \tag{15}$$

Employing this in Eq.(6), we have

$$[\sigma_{yz}(x,y,t)] = 0, \quad \text{for all } x \tag{16}$$

Define the Fourier transform by the equations

$$\bar{f}(s) = \int_{-\infty}^{\infty} f(x)e^{-isx}dx \tag{17}$$

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\bar{f}(s)e^{isx}ds \tag{18}$$

For solving the problem, the Fourier transform of Eq.(13) with respect to x can be given as follows:

$$\mu\int_{-h}^{h}\bar{\alpha}(|s|,|y-y|)\left[(-s^2)\bar{w}+\frac{\partial^2\bar{w}}{\partial y^2}\right]dy + \rho\omega^2\bar{w} = 0 \tag{19}$$

What should be done now is to solve the integrodifferential Eq.(19) for the function w. It seems obvious that a rigorous solution of such a problem has encountered serious if not unsurmountable mathematical difficulties, and one has to resort to an approximate procedure. In the given problem, according to the Refs.[10,11], the appropriate numerical procedure seems to follow naturally from the hypothesis of the attenuating neighbourhood underlying the theory of non-local continua. According to this hypothesis, the influence of the particle of the body on the thermoelectric state at the particle under observation, subsides rather rapidly with an increasing distance from the particle. In the classical theory, the function that characterizes the particle interaction is the Dirac delta function since in this theory the actions are assumed to have a zero range. In non-local theories the intermolecular forces may be represented by a variety of functions as long as their values decrease rapidly with the distance. In the present study, as adequate functions it was decided to select the terms, $\delta_n(y'-y)$, $n=1,2,\cdots$, of the so-called δ-sequences. A δ-sequence, as generally known, is (in the present case a one-dimensional) Dirac delta function, $\delta(y'-y)$. With respect to the terms of the adopted delta sequence, the following simplifying assumptions were accepted: (see the Refs.[10,11]). Nowinski has solved several non-local problems by using this kind of assumption.)

(a) For a sufficiently large j (as compared with the sphere of interactions of the particles), it is permissible to make the replacement

$$\int_{-j}^{j} f(y')\,\delta_n(y'-y)dy' \approx \int_{-\infty}^{\infty} f(y')\delta(y'-y)dy' \tag{20}$$

(b) As a consequence, the terms $\delta_n(y'-y)$, $n \gg 1$ acquire the shifting property of the Dirac function,

$$\int_{-j}^{j} f(y')\delta_n(y'-y)dy' \approx f(y) \tag{21}$$

The influence function was sought in the separable form. In this paper, the strip dynamic fracture problem is investigated. The cracks are parallel to the edges of the strip. The strip occupies the region $-\infty < x < \infty$, $|y| \leqslant h$. So according to the above discussion, the non-local interaction in y direction can be ignored. In view of our assumptions, we can give

$$\bar{\alpha}(|s|,|y'-y|) = \bar{\alpha}_0(s)\delta_n(y'-y) \tag{22}$$

From Eq.(19), it can be shown that

$$\frac{\partial^2\bar{w}}{\partial y^2} - \gamma^2\bar{w} = 0 \tag{23}$$

where $\gamma^2 = s^2 - \omega^2/[c^2\bar{\alpha}_0(s)]$, $c^2 = \mu/\rho$.

Because of symmetry, it suffices to consider the problem in the first quadrant only. The solution of the Eq.(23) does not present difficulties, it can be written as follows ($y \geqslant 0$):

$$\bar{w}(s,y,t) = A_1(s)\exp(-\gamma y) + B_1(s)\exp(\gamma y) \tag{24}$$

where $A_1(s)$ and $B_1(s)$ can be determined from the boundary conditions. The boundary conditions (8) - (12) can be applied to yield

$$\int_0^\infty A(s)\cos(sx)\mathrm{d}s = 0 \qquad x > l \tag{25}$$

$$\int_0^\infty \bar{\alpha}_0(s)\gamma[1-\exp(-2\gamma h)]/[1+\exp(-2\gamma h)]A(s)\cos(sx)\mathrm{d}s = \pi\tau_0/2\mu, \qquad 0 < x \leqslant l. \tag{26}$$

where $A(s) = [1+\exp(-2\gamma h)]A_1(s)$, $B_1(s) = \exp(-2\gamma h)A_1(s)$.

The Eqs.(25) and (26) are the dual integral equations of this problem.

V. SOLUTION OF THE DUAL INTEGRAL EQUATION

The non-local modulus α will depend on a characteristic length ratio a/l, where a is an internal characteristic length (e.g., lattice parameter, granular distance. In this paper, a represents lattice parameter.) and l is an external characteristic length (e.g., crack length, wave-length. In this paper, l represents the crack length.). By matching the dispersion curves of plane waves with those of atomic lattice dynamics (or experiments), we can determine the non-local modulus function α for a given material. Here, the only difference between the classical and non-local equations is in the introduction of the function $\bar{\alpha}_0(s)$ and it is logical to utilize the classical solution to convert the system Eqs.(25) and (26) into an integral equation of the second kind which is generally better behaved. If $\bar{\alpha}_0(s) = 1$ (the classical elastic case), Eqs.(25) and (26) reduce to the dual integral equations for the same problem in classical elasticity. Of course, the dual integral Eqs.(25) and (26) can be considered to be a single integral equation of the first kind with a discontinuous kernel (Eringen et al.[7]). It is well-known in the literature that integral equations of the first kind are generally ill-posed in the sense of Hadamard, e.g. small perturbations of the data can yield arbitrarily large changes in the solution. This makes the numerical solution of such equations quite difficult. In this paper, the Schmidt method (Morse, 1958) is used to overcome the difficulty. As discussed by Eringen et al.[7-9] and Nowinski[10,11], the following equations are assumed:

$$\alpha_0 = \chi_0\exp[-(\beta/a)^2(x'-x)^2] \tag{27}$$

$$\chi_0 = \beta/(a\sqrt{\pi}) \tag{28}$$

where β is a constant (here $\beta = e_0\sqrt{\pi}/l$, e_0 is a constant appropriate to each material.). a is the lattice parameter. So it can be found that

$$\bar{\alpha}_0(s) = \exp[-(sa)^2/(2\beta)^2] \tag{29}$$

and $\bar{\alpha}_0(s) = 1$ for the limit $a \to 0$, so that Eqs.(25) and (26) are reduced to the well-known equation of the classical theory. Here the Schmidt method[26] can be used to solve the dual integral Eqs.(25) and (26). The displacement w is presented by the following series:

$$w = \sum_{n=1}^{\infty} a_n P_{2n-2}^{(1/2,1/2)}(x/l)(1-x^2/l^2)^{1/2}, \quad \text{for} \quad |x| \leqslant l,\ y = 0 \tag{30}$$

$$w = 0, \qquad \text{for} \quad |x| > l,\ y = 0 \tag{31}$$

where a_n are unknown coefficients to be determined and $P_n^{(1/2,1/2)}(x)$ is a Jacobi polynomial[20]. The Fourier transformation of Eq.(30) is[21]

$$A(s) = \bar{w}(s,0,t) = \sum_{n=1}^{\infty} a_n G_n J_{2n-1}(ls)/s \tag{32}$$

$$G_n = 2\sqrt{\pi}(-1)^{n-1}\Gamma(2n-1/2)/(2n-2)! \tag{33}$$

Note that $\Gamma(x)$ and $J_n(x)$ are the Gamma and Bessel functions, respectively.

Substituting Eq.(32) into Eqs.(25) and (26), respectively, the Eq.(25) can be automatically satisfied, the Eq.(26) reduces to the form for $|x| \leqslant l$

$$\sum_{n=1}^{\infty} a_n G_n \int_0^{\infty} \bar{\alpha}_0(s)\gamma \frac{1-\exp(-2\gamma h)}{s[1+\exp(-2\gamma h)]} J_{2n-1}(ls)\cos(xs)\mathrm{d}s = \frac{\pi\tau_0}{2\mu} \tag{34}$$

The semi-infinite integral in Eq.(34) can be evaluated numerically by Filon method[22], except for singularities in the integrands of the integrals in Eq.(34). These singularities are poles that occur in the complex plane at the zero of the function $1+\exp(-2\gamma h)$, such as $2\gamma h = i\pi$, $3i\pi$, $5i\pi$, $\cdots$. All poles depend on the material, the incident wave frequency ω and the lattice parameter. It can be noted that the integral of Eq.(34) is not convergent at these poles. However, there is no pole for $\omega/c < \pi/(2h)$. So the integral of Eq.(34) is convergent at these poles for $\omega/c < \pi/(2h)$. In this paper, only the case of $\omega/c < \pi/(2h)$ is discussed. From Ref.[19], this case may be consistent with the statement that only shear waves with $\omega/c < \pi/(2h)$ can propagate in an elastic strip of width $2h$. This is in agreement with the well-known results that frequencies with which shear waves can propagate are less than a parameter depending on the width of the strip. As for $\omega/c > \pi/(2h)$, it should be further investigated. Equation (26) can now be solved for the coefficients a_n by the Schmidt method[16] for $\omega/c < \pi/(2h)$. For brevity, Eq.(34) can be rewritten as:

$$\sum_{n=1}^{\infty} a_n E_n(x) = U(x) \tag{35}$$

where $E_n(x)$ and $U(x)$ are known functions and coefficients a_n are unknown and will be determined. A set of functions $P_n(x)$ which satisfy the orthogonality condition

$$\int_0^l P_m(x)P_n(x)\mathrm{d}x = N_n\delta_{mn}, \qquad N_n = \int_0^l P_n^2(x)\mathrm{d}x \tag{36}$$

can be constructed from the function, $E_n(x)$, such that

$$P_n(x) = \sum_{i=1}^{n} M_{in}E_i(x)/M_{nn} \tag{37}$$

where M_{in} is the cofactor of the element d_{in} of D_n, which is defined as

$$D_n = \begin{bmatrix} d_{11}, d_{12}, d_{13}, \cdots, d_{1n} \\ d_{21}, d_{22}, d_{23}, \cdots, d_{2n} \\ d_{31}, d_{32}, d_{33}, \cdots, d_{3n} \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ d_{n1}, d_{n2}, d_{n3}, \cdots, d_{nn} \end{bmatrix}, \quad d_{ij} = \int_0^l E_i(x)E_j(x)\mathrm{d}x \tag{38}$$

Using Eqs.(35) – (37), we have

$$a_n = \sum_{j=n}^{\infty} q_j M_{nj}/M_{jj} \tag{39}$$

with
$$q_j = \frac{1}{N_j}\int_0^l U(x)P_j(x)\mathrm{d}x \tag{40}$$

Ⅵ. NUMERICAL CALCULATIONS AND DISCUSSION

When coefficients a_n are known, the entire dynamic stress field is obtained. However, in fracture mechanics, it is of importance to determine dynamic stress τ_{yz} along the crack line. τ_{yz} at $y = 0$ is given as follows:

$$\tau_{yz} = -\frac{2\mu}{\pi}\sum_{n=1}^{\infty} a_n G_n \int_0^{\infty} \bar{\alpha}_0 \gamma \frac{1-\exp(-2\gamma h)}{s[1+\exp(-2\gamma h)]} J_{2n-1}(ls)\cos(sx)\mathrm{d}s \tag{41}$$

For $a = 0$ at $x = \pm l$, it has the classical stress singularity. However, so long as $a \neq 0$, (41) gives a finite stress all along $y = 0$. At $-l < x < l$, τ_{yz}/τ_0 is very close to unity, and for $x > l$, τ_{yz}/τ_0 possesses finite values diminishing from a maximum value at $x = l$ to zero at $x = \infty$. Since $a/(2\beta l) > 1/100$ represents a crack length of less than 100 atomic distances (Eringen[8]), serious questions may arise regarding the interatomic arrangements and force laws, the solutions are not pursued at such crack sizes. The dynamic stress is computed numerically for the Lamé constants $\lambda = 98$ GPa, $\mu = 77$ GPa, $\rho = 7.7 \times 10^3$ (kg/m^3). The semi-infinite numerical integrals are evaluated easily by Filon and Simpson methods because of the rapid diminution of the integrands. From Refs. [23,24], it can be seen that the Schmidt method is performed satisfactorily if the first ten terms of infinite series to Eq. (35) are retained. Because the integrands of Eqs. (34) and (41) are complex, in all the figures, the shear stress along the crack face has a slight variation. In all computations, the limiting condition $\omega/c < \pi/(2h)$ is kept in view of the values chosen for ω/c and the strip width h. The results are plotted in Figs. 2 – 8.

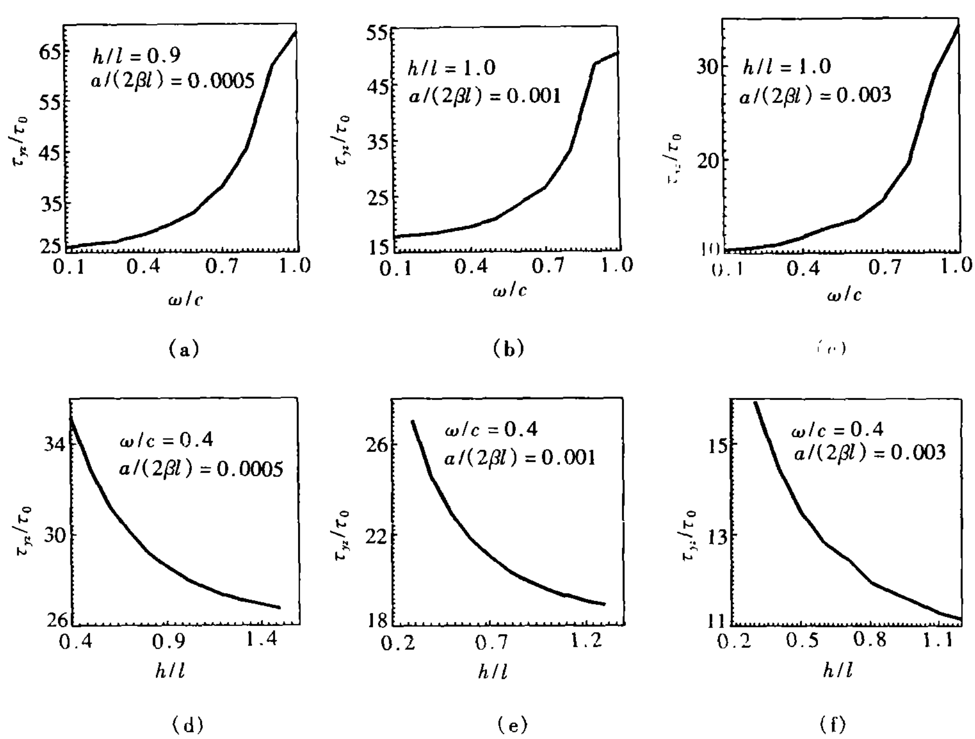

Fig. 2 The stress at the crack tip.

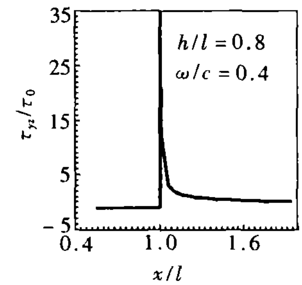

Fig.3 The variation of the stress along the crack line for $a/(2\beta l) = 0.0005$.

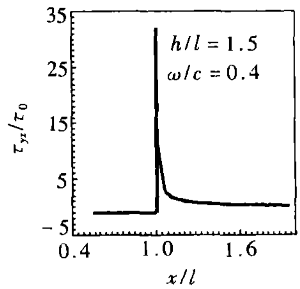

Fig.4 The variation of the stress along the crack line for $a/(2\beta l) = 0.0005$.

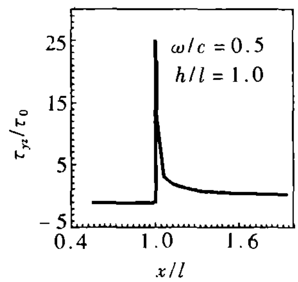

Fig.5 The variation of the stress along the crack line for $a/(2\beta l) = 0.001$.

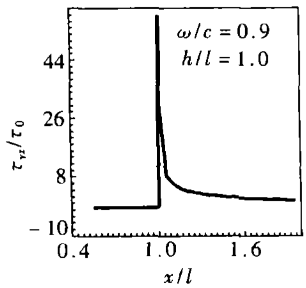

Fig.6 The variation of the stress along the crack line for $a/(2\beta l) = 0.001$.

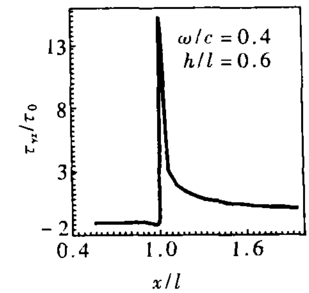

Fig.7 The variation of the stress along the crack line for $a/(2\beta l) = 0.003$.

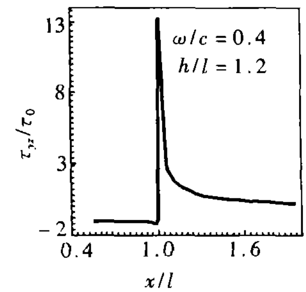

Fig.8 The variation of the stress along the crack line for $a/(2\beta l) = 0.003$.

The following observations can be made:

(1) The method used in this paper can overcome the mathematical difficulties that occur in Eringen's papers[7-9]. The results are more accurate than Eringen's and the method is more reasonable than his as well.

(2) The maximum stress does not occur at the crack tip, but slightly away from it. This phenomenon has been thoroughly substantiated by Eringen[25]. The maximum stress is finite. The distance between the crack tip and the maximum stress point is very small. Contrary to the classical elasticity solution, it is found that no stress singularity is present at the crack tip, and also the present results converge to the classical ones for positions far away from the crack tip.

(3) The dynamic shear stress at the crack tip becomes infinite as the atomic distance $a \to 0$. This is the classical continuum limit of square root singularity.

(4) For the a/β = constant, viz., the atomic distance does not change, the values of the dynamic stress concentration (at the crack tip) becomes higher with an increase of the crack length. It can be seen from experiments that materials with small cracks are more resistant to fracture than those with large ones.

(5) The dynamic shear stresses increase as the frequency ω becomes larger.

(6) The dynamic shear stresses decrease as the width of strip becomes larger.

(7) The significance of this result lies in that the fracture criteria are unified at both the macroscopic and microscopic scales, viz., it may solve the problem of a crack of any length.

(8) The present results converge to the classical ones when far away from the crack tip. However

they are different near the crack tip.

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