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INVESTIGATION OF ANTI-PLANE SHEAR BEHAVIOR OF TWO COLLINEAR CRACKS IN PIEZOELECTRIC MATERIALS BY A NEW METHOD*

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ABSTRACT In this paper, the interaction between two collinear cracks in piezoelectric materials under anti-plane shear loading was investigated for the impermeable crack face conditions. By using the Fourier transform, the problem can be solved with two pairs of triple integral equations. These equations are solved using Schmidt's method. This process is quite different from that adopted previously. This study makes it possible to understand the two collinear cracks interaction in piezoelectric materials.

KEY WORDS collinear crack, integral equation, Schmidt method

I. INTRODUCTION

It is well known that piezoelectric materials produce an electric field when deformed, and undergo deformation when subjected to an electric field. The coupling nature of piezoelectric materials has found wide application in electro-mechanical and electrical devices, such as electro-mechanical actuators, sensors and structures. When subjected to mechanical and electrical loads in service, these piezoelectric materials can fail prematurely due to their brittleness and presence of defects or flaws produced during their manufacturing process. Therefore, it is important to study the electro-elastic interaction and fracture behaviors of piezoelectric materials.

Many studies have been made on the electro-elastic fracture mechanics based on the modeling and analysis of one crack in the piezoelectric materials. (see, for examples, Deeg, 1980; Pak, 1990, 1992; Sosa, 1992; Suo et al., 1992; Suo, 1993; Park and Sun, 1995a, b; Zhang and Tong, 1996; Zhang et al., 1998; Gao et al., 1997; Wang, 1992). Most recently, Chen and Karihaloo (1999) considered an infinite piezoelecric ceramic with impermeable crack-face boundary condition under arbitrary electro-mechanical impact. Chen and Yu (1999) studied the transient response of a piezoelectric ceramic with coplanar cracks under electromechanical impact for impermeable boundary conditions by using the Fourier integral transform and the singular integral equation method. Some significant results have been obtained in Chen and Yu's (1999) paper. Sosa and Hhutoryansky (1999) investigated the response of piezoelectric bodies disturbed by internal electric sources. The impermeable boundary condition on the crack surface was widely used in the investigations (Pak, 1990, 1992; Suo et al.,

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1992; Suo, 1993; Park and Sun, 1995a, 1995b; Chen and Karihaloo, 1999, Chen and Yu, 1999). The problem of the interacting fields among multiple cracks in a piezoelectric material was studied by Han (Han et al., 1999). In Han's paper, the crack is treated as continuous distributed dislocations with the density function to be determined according to the conditions of external loads and the crack surface. However, the problem of the collinear cracks in piezoelectric materials was not studied in Han's paper. He just gave some special cases as examples.

In the present paper, the interaction between two collinear symmetrical impermeable cracks subjected to anti-plane shear in piezoelectric materials was investigated using a somewhat different approach, namely, Schmidt's method (Morse et al., 1958). It is a simple and convenient method for solving this problem. Fourier transform is applied and a mixed boundary value problem is reduced to two pairs of triple integral equations. In solving the triple integral equations, the crack surface displacement and electric potential are expanded in a series using Jacobi's polynomials. This process is quite different from that adopted in references (Han et al., 1999; Deeg, 1980; Pak, 1990, 1992; Sosa, 1992; Suo et al., 1992; Park and Sun, 1995a,b; Zhang and Tong, 1996; Zhang et al., 1998; Gao et al., 1997; Wang, 1992; Chen and Karihaloo, 1999; Sosa and Hhutoryansky, 1999; Chen and Yu, 1999). The form of solution is easy to understand. Numerical calculations are carried out for the stress intensity factors and the electric displacement intensity factors.

Ⅱ. FORMULATION OF THE PROBLEM

Consider an infinite piezoelectric body containing two collinear symmetric impermeable cracks of length $(1-b)$ along the x-axis. $2b$ is the distance between the two cracks. The piezoelectric boundary-value problem for anti-plane shear is considerably simplified if we consider only the out-of-plane displacement and the in-plane electric fields. The plate is subjected to a constant stress $\tau_{yz}=-\tau_0$, and a constant electric displacement $D_y=-D_0$ along the surface of the cracks, see Fig.1, such that the constitutive equations can be written as

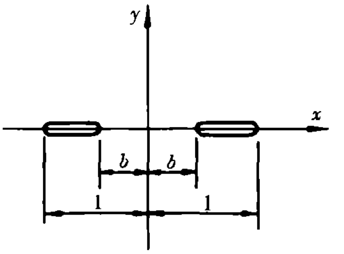

Fig.1 Cracks in a piezoelectric materials body under anti-plane shear.

$$\tau_{zk}=c_{44}w_{,k}+e_{15}\phi_{,k},\quad D_k=e_{15}w_{,k}-\varepsilon_{11}\phi_{,k} \tag{1,2}$$

where τ_{zk}, $D_k(k=x,y)$ are the anti-plane shear stress and in-plane electric displacement, respectively. c_{44}, e_{15}, ε_{11} are the shear modulus, piezoelectric coefficient and dielectric parameter, respectively. w and ϕ are the mechanical displacement and electric potential.

The anti-plane governing equations for piezoelectric materials are (Shindo, Narita and Tanaka, 1996)

$$c_{44}\nabla^2 w+e_{15}\nabla^2\phi=0 \tag{3}$$

$$e_{15}\nabla^2 w-\varepsilon_{11}\nabla^2\phi=0 \tag{4}$$

where $\nabla^2=\partial^2/\partial x^2+\partial^2/\partial y^2$ is the two-dimensional Laplace operator. Body force, other than inertia, and the free charge are ignored in the present work. Because of the assumed symmetry in geometry and loading, it is sufficient to consider the problem for $0\leqslant x\leqslant\infty$, $0\leqslant y\leqslant\infty$ only.

A Fourier transform is applied to Eqs.(3) and (4). Assume that the solution is

$$\bar{w}(s,y)=A(s)e^{-sy} \tag{5}$$

where $A(s)$ is an unknown function and the superposed bar indicates the Fourier transform throughout the paper, e.g.,

$$\bar{f}(s) = \int_{-\infty}^{\infty} f(x) e^{-isx} dx \tag{6}$$

Inserting Eq.(5) into Eq.(4), it can be assumed that

$$\bar{\phi}(s,y) - \frac{e_{15}}{\varepsilon_{11}} \bar{w}(s,y) = B(s) e^{-sy} \tag{7}$$

where $B(s)$ is an unknown function.

As discussed in Narita's (Narita and Shindo, 1998), Shindo's (Shindo, 1996) and Yu's (Yu and Chen 1998) references, the impermeable boundary conditions of the present problem are:

$$\tau_{yz}(x,0) = -\tau_0, \qquad b \leqslant |x| \leqslant 1 \tag{8}$$

$$D_y(x,0) = -D_0, \qquad b \leqslant |x| \leqslant 1 \tag{9}$$

$$w(x,0) = \phi(x,0) = 0, \qquad |x| < b, \ |x| > 1 \tag{10}$$

$$w(x,y) = \phi(x,y) = 0, \qquad \text{for} (x^2 + y^2)^{1/2} \to \infty \tag{11}$$

The problem therefore reduces to the determination of the two unknown function $A(s)$ and $B(s)$. Because of symmetry, the boundary conditions can be applied to yield two pairs of triple integral equations:

$$\frac{2}{\pi}\int_0^{\infty} A(s)\cos(sx) ds = 0 \qquad 0 \leqslant x < b \text{ and } x > 1 \tag{12}$$

$$\frac{2}{\pi}\int_0^{\infty} sA(s)\cos(sx) ds = \frac{1}{\mu}\left(\tau_0 + \frac{e_{15} D_0}{\varepsilon_{11}}\right) \qquad b \leqslant x \leqslant 1 \tag{13}$$

and

$$\frac{2}{\pi}\int_0^{\infty} B(s)\cos(sx) ds = 0 \qquad 0 \leqslant x < b \text{ and } x > 1 \tag{14}$$

$$\frac{2}{\pi}\int_0^{\infty} sB(s)\cos(sx) ds = -\frac{D_0}{\varepsilon_{11}} \qquad b \leqslant x \leqslant 1 \tag{15}$$

where $\mu = c_{44} + \frac{e_{15}^2}{\varepsilon_{11}}$.

To determine the unknown functions $A(s)$, $B(s)$, the above two pairs of triple integral equations (12, 13, 14, 15) should be solved.

Ⅲ. SOLUTION OF THE TRIPLE INTEGRAL EQUATION

To solve the above two pairs of triple integral equations (12 – 15), the Schmidt's method (Morse et al., 1958) can be used. The displacement w and the electric potential ϕ can be represented by the following series:

$$w(x,0) = \sum_{n=0}^{\infty} a_n P_n^{(1/2,1/2)}\left(\frac{x - \frac{1+b}{2}}{\frac{1-b}{2}}\right)\left(1 - \frac{\left(x - \frac{1+b}{2}\right)^2}{\left(\frac{1-b}{2}\right)^2}\right)^{\frac{1}{2}}, \text{for } b \leqslant x \leqslant 1,\ y = 0 \tag{16}$$

$$w(x,0) = 0, \qquad \text{for } x > 1,\ x < b,\ y = 0 \tag{17}$$

$$\phi(x,0) = \sum_{n=0}^{\infty} b_n P_n^{(1/2,1/2)}\left(\frac{x - \frac{1+b}{2}}{\frac{1-b}{2}}\right)\left(1 - \frac{\left(x - \frac{1+b}{2}\right)^2}{\left(\frac{1-b}{2}\right)^2}\right)^{\frac{1}{2}}, \text{for } b \leqslant x \leqslant 1,\ y = 0 \tag{18}$$

$$\phi(x,0) = 0, \qquad \text{for } x > 1,\ x < b,\ y = 0 \tag{19}$$

where a_n and b_n are unknown coefficients to be determined and $P_n^{(1/2,1/2)}(x)$ is a Jacobi polynomial (Gradshteyn and Ryzhik, 1980). The Fourier transformation of Eqs. (16) and (18) is (Erdelyi, 1954)

$$A(s) = \bar{w}(s,0) = \sum_{n=0}^{\infty} a_n B_n G_n(s) \frac{1}{s} J_{n+1}\left(s \frac{1-b}{2}\right) \tag{20}$$

$$B(s) = \bar{\phi}(s,0) - \frac{e_{15}}{\varepsilon_{11}} \bar{w}(s,0) = \sum_{n=0}^{\infty}\left(b_n - \frac{e_{15}}{\varepsilon_{11}} a_n\right) B_n G_n(s) \frac{1}{s} J_{n+1}\left(s \frac{1-b}{2}\right) \tag{21}$$

$$B_n = 2\sqrt{\pi} \frac{\Gamma\left(n+1+\frac{1}{2}\right)}{n!} \tag{22}$$

$$G_n(s) = \begin{cases} (-1)^{n/2} \cos\left(s \frac{1+b}{2}\right), & n = 0,2,4,6,\cdots \\ (-1)^{(n+1)/2} \sin\left(s \frac{1+b}{2}\right), & n = 1,3,5,7,\cdots \end{cases} \tag{23}$$

where $\Gamma(x)$ and $J_n(x)$ are the Gamma and Bessel functions, respectively.

By using Eqs. (20) and (21), Eqs. (12) and (14) can be automatically satisfied, and the remaining Eqs. (13) and (15) reduce to the forms after integration with respect to x in $[b, x]$, respectively.

$$\sum_{n=0}^{\infty} a_n B_n \int_0^{\infty} s^{-1} G_n(s) J_{n+1}\left(s \frac{1-b}{2}\right)[\sin(sx) - \sin(sb)] \mathrm{d}s = \frac{\pi}{2\mu} \tau_0 (1+\lambda)(x-b) \tag{24}$$

$$\sum_{n=0}^{\infty}\left(b_n - \frac{e_{15}}{\varepsilon_{11}} a_n\right) B_n \int_0^{\infty} s^{-1} G_n(s) J_{n+1}\left(s \frac{1-b}{2}\right)[\sin(sx) - \sin(sb)] \mathrm{d}s = -\frac{\pi D_0}{2\varepsilon_{11}}(x-b) \tag{25}$$

where $\lambda = \frac{e_{15} D_0}{\varepsilon_{11} \tau_0}$.

From Eqs. (24) and (25), it can be shown that the unknown coefficients a_n and b_n have the following relationship:

$$b_n = \left(\frac{e_{15}}{\varepsilon_{11}} - \frac{D_0 \mu}{\varepsilon_{11} T_0}\right) a_n, \qquad T_0 = \tau_0(1+\lambda) \tag{26}$$

So it suffices to solve Eq. (24) for the present problem. The semi-infinite integral in equation (24) can be modified as (Gradshteyn and Ryzhik, 1980)

$$\int_0^{\infty} \frac{1}{s} J_{n+1}\left(s \frac{1-b}{2}\right) \cos\left(s \frac{1+b}{2}\right) \sin(sx) \mathrm{d}s = \frac{1}{2(n+1)}\left\{\frac{\left(\frac{1-b}{2}\right)^{n+1} \sin\left[\frac{(n+1)\pi}{2}\right]}{\left[x + \frac{1+b}{2} + \sqrt{\left(x + \frac{1+b}{2}\right)^2 - \left(\frac{1-b}{2}\right)^2}\right]^{n+1}} - \sin\left[(n+1)\sin^{-1}\left(\frac{1+b-2x}{1-b}\right)\right]\right\} \tag{27}$$

$$\int_0^{\infty} \frac{1}{s} J_{n+1}\left(s \frac{1-b}{2}\right) \sin\left(s \frac{1+b}{2}\right) \sin(sx) \mathrm{d}s = \frac{1}{2(n+1)}\left\{\cos\left[(n+1)\sin^{-1}\left(\frac{1+b-2x}{1-b}\right)\right] - \frac{\left(\frac{1-b}{2}\right)^{n+1} \cos\left[\frac{(n+1)\pi}{2}\right]}{\left[x + \frac{1+b}{2} + \sqrt{\left(x + \frac{1+b}{2}\right)^2 - \left(\frac{1-b}{2}\right)^2}\right]^{n+1}}\right\} \tag{28}$$

Thus the semi-infinite integral in Eq.(24) can be evaluated directly. Equation (24) can now be solved for the coefficients a_n by the Schmidt's method (Morse et al., 1958). For brevity, Eq.(24) is rewritten as

$$\sum_{n=0}^{\infty} a_n E_n(x) = U(x), \qquad b < x < 1 \tag{29}$$

where $E_n(x)$ and $U(x)$ are known functions and coefficients a_n are unknown and will be determined. A set of functions $P_n(x)$ which satisfy the orthogonality condition

$$\int_b^1 P_m(x) P_n(x) \mathrm{d}x = N_n \delta_{mn}, \qquad N_n = \int_b^1 P_n^2(x) \mathrm{d}x \tag{30}$$

can be constructed from the function, $E_n(x)$, such that

$$P_n(x) = \sum_{i=0}^{n} \frac{M_{in}}{M_{nn}} E_i(x) \tag{31}$$

where M_{in} is the cofactor of the element d_{in} of D_n, which is defined as

$$D_n = \begin{bmatrix} d_{00}, d_{01}, d_{02}, \cdots, d_{0n} \\ d_{10}, d_{11}, d_{12}, \cdots, d_{1n} \\ d_{20}, d_{21}, d_{22}, \cdots, d_{2n} \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ d_{n0}, d_{n1}, d_{n2}, \cdots, d_{nn} \end{bmatrix}, \quad d_{ij} = \int_b^1 E_i(x) E_j(x) \mathrm{d}x \tag{32}$$

Using Eqs.(29) - (32), we obtain

$$a_n = \sum_{j=n}^{\infty} q_j \frac{M_{nj}}{M_{jj}} \tag{33}$$

with

$$q_j = \frac{1}{N_j} \int_b^1 U(x) P_j(x) \mathrm{d}x \tag{34}$$

Ⅳ. STRESS INTENSITY FACTORS AND ELECTRIC DISPLACEMENT

When the coefficients a_n and b_n are known, the entire stress field and the electric displacement can be obtained. However, in fracture mechanics, it is of importance to determine stress τ_{yz} and the electric displacement D_y in the vicinity of the crack tip. τ_{yz} and D_y along the crack line can be expressed respectively as

$$\begin{aligned} \tau_{yz}(x,0) &= -\frac{2}{\pi} \sum_{n=0}^{\infty} (c_{44} a_n + e_{15} b_n) B_n \int_0^{\infty} G_n(s) J_{n+1}\left(s \frac{1-b}{2}\right) \cos(xs) \mathrm{d}s \\ &= -\frac{2\mu}{\pi(1+\lambda)} \sum_{n=0}^{\infty} a_n B_n \int_0^{\infty} G_n(s) J_{n+1}\left(s \frac{1-b}{2}\right) \cos(xs) \mathrm{d}s \end{aligned} \tag{35}$$

$$\begin{aligned} D_y(x,0) &= -\frac{2}{\pi} \sum_{n=0}^{\infty} (e_{15} a_n - \varepsilon_{11} b_n) B_n \int_0^{\infty} G_n(s) J_{n+1}\left(s \frac{1-b}{2}\right) \cos(xs) \mathrm{d}s \\ &= -\frac{2D_0\mu}{\pi T_0} \sum_{n=0}^{\infty} a_n B_n \int_0^{\infty} G_n(s) J_{n+1}\left(s \frac{1-b}{2}\right) \cos(xs) \mathrm{d}s \end{aligned} \tag{36}$$

Observing the expression in Eqs.(35) and (36), the singular portion of the stress field and that of the electric displacement can be obtained respectively from the relationships (Gradshteyn and Ryzhik, 1980)

$$\cos\left(s\frac{1+b}{2}\right)\cos(sx) = \frac{1}{2}\left\{\cos\left[s\left(\frac{1+b}{2}-x\right)\right]+\cos\left[s\left(\frac{1+b}{2}+x\right)\right]\right\}$$

$$\sin\left(s\frac{1+b}{2}\right)\cos(sx) = \frac{1}{2}\left\{\sin\left[s\left(\frac{1+b}{2}-x\right)\right]+\sin\left[s\left(\frac{1+b}{2}+x\right)\right]\right\}$$

$$\int_0^\infty J_n(sa)\cos(bs)\mathrm{d}s = \begin{cases} \dfrac{\cos[n\sin^{-1}(b/a)]}{\sqrt{a^2-b^2}}, & a > b \\ -\dfrac{a^n\sin(n\pi/2)}{\sqrt{b^2-a^2}[b+\sqrt{b^2-a^2}]^n}, & b > a \end{cases}$$

$$\int_0^\infty J_n(sa)\sin(bs)\mathrm{d}s = \begin{cases} \dfrac{\sin[n\sin^{-1}(b/a)]}{\sqrt{a^2-b^2}}, & a > b \\ \dfrac{a^n\cos(n\pi/2)}{\sqrt{b^2-a^2}[b+\sqrt{b^2-a^2}]^n}, & b > a \end{cases}$$

The singular portion of the stress field and that of the electric displacement can be expressed respectively as follows

$$\tau = -\frac{\mu}{\pi(1+\lambda)}\sum_{n=0}^{\infty} a_n B_n H_n(b,x) \tag{37}$$

$$D = -\frac{D_0\mu}{\pi T_0}\sum_{n=0}^{\infty} a_n B_n H_n(b,x) \tag{38}$$

where

$$H_n(b,x) = -F_1(b,x,n) \qquad n = 0,1,2,3,4,5,\cdots \ (\text{for } 0 < x < b)$$

$$H_n(b,x) = (-1)^{n+1}F_2(b,x,n), \qquad n = 0,1,2,3,4,5,\cdots, \ (\text{for } 1 < x)$$

$$F_1(b,x,n) = \frac{2(1-b)^{n+1}}{\sqrt{(1+b-2x)^2-(1-b)^2}[1+b-2x+\sqrt{(1+b-2x)^2-(1-b)^2}]^{n+1}}$$

$$F_2(b,x,n) = \frac{2(1-b)^{n+1}}{\sqrt{(2x-1-b)^2-(1-b)^2}[2x-1-b+\sqrt{(2x-1-b)^2-(1-b)^2}]^{n+1}}$$

At the left end of the right crack, we obtain the stress intensity factor K_L as

$$K_L = \lim_{x\to b^-}\sqrt{2\pi(b-x)}\cdot\tau = \frac{\mu}{1+\lambda}\sqrt{\frac{2}{\pi(1-b)}}\sum_{n=0}^{\infty} a_n B_n \tag{39}$$

At the right end of the right crack, we obtain the stress intensity factor K_R as

$$K_R = \lim_{x\to 1^+}\sqrt{2\pi(x-1)}\cdot\tau = \frac{\mu}{1+\lambda}\sqrt{\frac{2}{\pi(1-b)}}\sum_{n=0}^{\infty}(-1)^n a_n B_n \tag{40}$$

At the left end of the right crack, we obtain the electric displacement intensity factor D_L as

$$D_L = \lim_{x\to b^-}\sqrt{2\pi(b-x)}\cdot D = \frac{D_0\mu}{T_0}\sqrt{\frac{2}{\pi(1-b)}}\sum_{n=0}^{\infty} a_n B_n = \frac{(1+\lambda)D_0}{T_0}K_L = \frac{D_0}{\tau_0}K_L \tag{41}$$

At the right end of the right crack, we obtain the electric displacement intensity factor D_R as

$$D_R = \lim_{x\to 1^+}\sqrt{2\pi(x-1)}\cdot D = \frac{D_0\mu}{T_0}\sqrt{\frac{2}{\pi(1-b)}}\sum_{n=0}^{\infty}(-1)^n a_n B_n = \frac{(1+\lambda)D_0}{T_0}K_R = \frac{D_0}{\tau_0}K_R \tag{42}$$

Ⅴ. NUMERICAL CALCULATIONS AND DISCUSSION

The dimensionless stress intensity factors K_L and K_R are calculated numerically. So the intensity factors D_L and D_R can be obtained from the relationships (41) and (42) by giving the electric loads and the stress loads. Adopting the first ten terms of the infinite series to equation (29), we performed the Schmidt procedure. For a check of the accuracy, the values of $\sum_{n=0}^{9} a_n E_n(x)$ and $U(x)$ are given in Table 1 for $b = 0.5$. In Table 2, the values of the coefficients a_n are given for $b = 0.5$. From the above results and references (see e.g. Itou, 1978, 1979; Zhou, 1999a, 1999b), it can be seen that the Schmidt's method is performed satisfactorily if the first ten terms of the infinite series to equation (29) are obtained. The behavior of the solution stays steady with an increase of the number of terms in Eq.(29). Hence, it is clear that the Schmidt's method is carried out satisfactorily. The precision of the present paper's solution can satisfy the demands of the practical problem. In no computation are the

Table 1 Values of $\sum_{n=0}^{9}(a_n E_n(x))\Big/\left[\frac{\pi\tau_0(1+\lambda)}{2\mu}\right]$ and $U(x)\Big/\left[\frac{\pi\tau_0(1+\lambda)}{2\mu}\right] = x - b$ for $b = 0.5$

x	$\sum_{n=0}^{9} \frac{a_n E_n(x)}{\frac{\pi\tau_0(1+\lambda)}{2\mu}}$	$\frac{U(x)}{\frac{\pi\tau_0(1+\lambda)}{2\mu}} = x - b$
0.5	0.0000	0.0000
0.6	0.1001	0.1000
0.7	0.1998	0.2000
0.8	0.2999	0.3000
0.9	0.4001	0.4000

Table 2 Values of $\frac{a_n}{\frac{\pi\tau_0(1+\lambda)}{2\mu}}$ for $b = 0.5$

n	$\frac{a_n}{\frac{\pi\tau_0(1+\lambda)}{2\mu}}$
0	0.161498E + 00
1	0.267988E - 03
2	0.277891E - 04
3	0.276478E - 05
4	0.268628E - 06
5	0.265218E - 07
6	0.432511E - 08
7	0.250341E - 09
8	0.348761E - 10
9	0.321782E - 11

Table 3 Variation with b of the stress intensity factors K_L and K_R, and of the electric displacement intensity factors D_L and D_R for $D_0/\tau_0 = 0.5$

b	K_L/τ_0	K_R/τ_0	D_L	D_R
0.01	2.81459	1.40362	1.40729	0.70181
0.02	2.31594	1.40617	1.15797	0.70309
0.03	2.06550	1.39318	1.03275	0.69659
0.06	1.70631	1.34613	0.85316	0.67307
0.10	1.49154	1.29063	0.74577	0.64532
0.15	1.34416	1.23151	0.67208	0.61576
0.20	1.24704	1.17887	0.62352	0.58944
0.30	1.11018	1.08274	0.55509	0.54137
0.40	1.00209	0.990704	0.50105	0.495352
0.50	0.901839	0.897294	0.450919	0.448647
0.60	0.799957	0.798319	0.399978	0.399159
0.70	0.689442	0.688957	0.34471	0.344478
0.80	0.561420	0.561323	0.280710	0.280661
0.90	0.396474	0.396467	0.198237	0.198233

material constants considered because the intensity factors do not depend on the material constants. The solution of two collinear cracks of arbitrary length $a - b$ can easily be obtained by a simple change in the numerical values of the present paper ($a > b > 0$), i.e., it can use the results of the collinear cracks of length $1 - b/a$ in the present paper. The solution of this paper is applicable for two collinear cracks of arbitrary length. However, the method in this paper is not valid for the two collinear cracks of different lengths. This should be further investigated. The results of the present paper are shown in Table 3. It can be seen that the stress intensity factors at the inner crack tips are bigger than those at the outer crack tips. The effects of the two collinear cracks decrease when the distance between the two collinear cracks increases.

Ⅵ. CONCLUSIONS

The anti-plane electro-elastic problem of a piezoelectric material with two collinear impermeable cracks has been analyzed theoretically. The traditional concept of linear elastic fracture mechanics is extended to include the piezoelectric effects and the results are expressed in terms of the stress intensity factors. The stress intensity factors increase when the distance between the two collinear cracks decreases. The stress intensity factors are found to be independent of the electrical loads and the material constants while dependent on the length of the crack. However, the intensity factors of the electric displacement are found to depend on the stress loads, the electrical loads and the stress intensity factors. They are also found to be independent of the material constants.

REFERENCES

[1] Amemiya, A. and Taguchi, T., Numerical Analysis and Fortran, Maruzen, Tokyo, 1969.

[2] Chen, Z.T., Karihaloo, B.L., Dynamic response of a cracked piezoelectric ceramic under arbitrary electro-mechanical impact, *International Journal of Solids and Structures*, Vol.36, 1999, 5125 - 5133.

[3] Chen, Z.T. and Yu, S.W., Transient response of a piezoelectric ceramic with coplanar cracks under electromechanical impact, *Acta Mechanica Sinica*, Vol.15, No.4, 1999, 327 - 333.

[4] Deeg, W.E.F., The analysis of dislocation, crack and inclusion problems in piezoelectric solids, Ph.D.Thesis, Stanford University, 1980.

[5] Erdelyi, A.(ed), Tables of Integral Transforms, McGraw-Hill, Vol.1, 1954, New York.

[6] Gao, H., Zhang, T.Y. and Tong, P., Local and global energy rates for an elastically yielded crack in piezoelectric ceramics, *Journal of Mechanics and Physics of Solids*, Vol.45, 1997, 491 - 510.

[7] Gradshteyn, I.S. and Ryzhik, I.M., Table of Integral, Series and Products, Academic Press, New York, 1980.

[8] Han, Xueli and Wang Tzuchiang, Interacting multiple cracks in piezoelectric materials, *International Journal of Solids and Structures*, Vol.36, 1999, 4183 - 4202.

[9] Itou, S., Three-dimensional waves propagation in a cracked elastic solid, *ASME Journal of Applied Mechanics*, Vol.45, 1978, 807 - 811.

[10] Itou, S., Three-dimensional problem of a running crack, *International Journal of Engineering Science*, Vol.17, 1979, 59 - 71.

[11] Morse, P.M. and Feshbach, H., Methods of Theoretical Physics, McGraw-Hill, New York, Vol.1, 1958.

[12] Narita, K. and Shindo, Y., Anti-plane shear crack growth rate of piezoelectric ceramic body with finite width, *Theoretical and Applied Fracture Mechanics*, Vol.30, 1998, 127 - 132.

[13] Pak, Y.E., Crack extension force in a piezoelectric material, *Journal of Applied Mechanics*, Vol.57, 1990, 647 - 653.

[14] Pak, Y.E., Linear electro-elastic fracture mechanics of piezoelectric materials, *International Journal of Fracture*, Vol.54, 1992, 79 - 100.

[15] Park, S.B. and Sun, C.T., Effect of electric field on fracture of piezoelectric ceramics, *International Journal of*

Fracture, Vol.70, 1995a, 203 - 216.

[16] Park, S. B. and Sun, C. T., Fracture criteria for piezoelectric ceramics, *Journal of American Ceramics Society*, Vol.78, 1995b, 1475 - 1480.

[17] Shindo, Y., Narita, K. and Tanaka, K., Electroelastic intensification near anti-plane shear crack in orthotropic piezoelectric ceramic strip, *Theoretical and Applied Fracture Mechanics*, Vol.25, 1996, 65 - 71.

[18] Sosa, H., On the fracture mechanics of piezoelectric solids, *International Journal of Solids and Structures*, Vol. 29, 1992, 2613 - 2622.

[19] Sosa, H. and Khutoryansky, N., Transient dynamic response of piezoelectric bodies subjected to internal electric impulses, *International Journal of Solids and Structures*, Vol.36, 1999, 5467 - 5484.

[20] Suo, Z., Models for breakdown-resistant dielectric and ferroelectric ceramics, *Journal of Mechanics and Physics of Solids*, Vol.41, 1993, 1155 - 1176.

[21] Suo, Z., Kuo, C.M., Barnett, D.M. and Willis, J.R., Fracture mechanics for piezoelectric ceramics, *Journal of Mechanics and Physics of Solids*, Vol.40, 1992, 739 - 765.

[22] Wang, B., Three-dimensional analysis of a flat elliptical crack in a piezoelectric material, *International Journal of Engineering Science*, Vol.30, No.6, 1992, 781 - 791.

[23] Yu, S. W. and Chen, Z. T., Transient response of a cracked infinite piezoelectric strip under anti-plane impact, *Fatigue and Engineering Materials and Structures*, Vol.21, 1998, 1381 - 1388.

[24] Zhang, T. Y. and Tong, P., Fracture mechanics for a mode Ⅲ crack in a piezoelectric material, *International Journal of Solids and Structures*, Vol.33, 1996, 343 - 359.

[25] Zhang, T. Y., Qian, C. F. and Tong, P., Linear electro-elastic analysis of a cavity or a crack in a piezoelectric material, *International Journal of Solids and Structures*, Vol.35, 1998, 2121 - 2149.

[26] Zhou, Z. G., Han, J. C. and Du, S. Y., Investigation of a Griffith crack subjected to anti-plane shear by using the non-local theory, *International Journal of Solids and Structures*, Vol.36, 1999a, 3891 - 3901.

[27] Zhou, Z. G., Han, J. C. and Du, S. Y., Two collinear Griffith cracks subjected to uniform tension in infinitely long strip, *International Journal of Solids and Structures*, Vol.36, 1999b, 5597 - 5609.