Pergamon

Mechanics Research Communications, Vol. 26, No. 4, pp. 415–420, 1999
Copyright © 1999 Elsevier Science Ltd
Printed in the USA. All rights reserved
0093-6413/99/$–see front matter

PII S0093-6413(99)00042-7

A SCREW DISLOCATION INTERACTING WITH A PIEZOELECTRIC BIMATERIAL INTERFACE

Jinxi Liu[+], *Shanyi Du*[++] *and Biao Wang*[++]
\+ *Department of Civil Engineering, Shijiazhuang Railway Institute, Shijiazhuang, 050043, P.R.China*
\++ *Center for Composite Materials, Harbin Institute of Technology, Harbin, 150001, P.R.China*

(Received 20 April 1998; accepted for print 19 July 1999)

Introduction

The interactions between dislocations and boundaries or interfaces of materials have been studied extensively for many years, mainly because it plays an important role in understanding the physical behavior of materials. Head[1] first investigated the interaction of a screw dislocation with a bimaterial interface. By calculating the image force acting on the screw dislocation, he found that the interface always repels the screw dislocation located in the material with a lower shear modulus. Dundurs and Sendeckyj[2] analyzed the interaction between an edge dislocation and a bimaterial interface. Barnett and Lothe[3] considered a dislocation with arbitrary Burgers vector in an anisotropic half-space. Recently, Ting and Barnett[4] computed the image force on line dislocations in anisotropic elastic half-spaces with a free or fixed boundary. They concluded that the free boundary always attracts the dislocation, while the fixed boundary repels it. More recently, Fan and Xiao[5] studied the interaction between a screw dislocation and a slightly wavy interface. The aforementioned works are limited to the purely elastic materials. When materials possess piezoelectric behavior, how do dislocations interact with boundaries or interfaces? Generalizing Eshelby's method of calculating the force acting on a dislocation in the purely elastic materials[6], Pak[7] derived the formula for computing the force on a piezoelectric dislocation in a piezoelectric material, where the piezoelectric screw dislocation includes the traditional elastic dislocation and the "electric-potential-dislocation"[8]. He also analyzed the interaction between a piezoelectric screw dislocation and a traction- charge-free boundary. He showed that the presence of piezoelectric effect can either increase or decrease the image force acting on a piezoelectric screw dislocation

In this paper, we consider the electroelastic interaction of a piezoelectric screw dislocation with a piezoelectric bimaterial interface. The main objective is to reveal the degree of influence of electro-mechanical coupling behavior on the image force on a screw dislocation.

Statement of the problem and its solution

Consider a piezoelectric composite which consists of two dissimilar transversely isotropic piezoelectric media with respect to the poling direction (z-direction), as shown in Fig.1. The regions occupied by the medium I and the medium II are referred to as S_1 and S_2, respectively. A piezoelectric screw dislocation $\boldsymbol{b} = \{b_3, b_\varphi\}^{\mathrm{T}}$ is located at $z_0 = x_0 + iy_0$ in the medium I, where b_φ is known as the electric-potential-dislocation and $i = \sqrt{-1}$. With the statement above, the out-of-plane displacement w and the electric potential φ are only functions of the variable x and y, such that $w = w(x, y)$, $\varphi = \varphi(x, y)$.

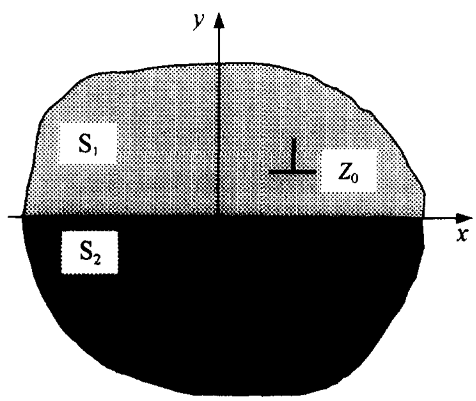

Fig.1 A piezoelectric screw dislocation in a piezoelectric bimaterial

Following Ref.[8], w and φ satisfy the governing equation

$$\nabla^2 \mathbf{u} = 0 \tag{1}$$

where ∇^2 is the two-dimensional Laplacian operator and $\mathbf{u} = \{w, \varphi\}^{\mathrm{T}}$, referred to as the generalized displacement vector.

For linear piezoelectric materials, the stresses and electric displacements can be expressed as

$$\mathbf{t}_x = \{\sigma_{zx}, D_x\}^{\mathrm{T}} = \mathbf{C}\frac{\partial \mathbf{u}}{\partial x}, \quad \mathbf{t}_y = \{\sigma_{zy}, D_y\}^{\mathrm{T}} = \mathbf{C}\frac{\partial \mathbf{u}}{\partial y} \tag{2}$$

with

$$\mathbf{C} = \begin{bmatrix} C_{44} & e_{15} \\ e_{15} & -\varepsilon_{11} \end{bmatrix} \tag{3}$$

where D denotes the electric displacement; C_{44}, e_{15} and ε_{11} are shear, piezoelectric and dielectric moduli, respectively.

Equation (1) indicates that $\mathbf{u}$ is a harmonic function vector which can be taken as the real part of some complex potentials of the complex variable $z = x + iy$, such that

$$\mathbf{u} = \mathrm{Re}\{f_1(z), f_2(z)\}^{\mathrm{T}} = \mathrm{Re}[\mathbf{f}(z)] \tag{4}$$

where Re stands for the real part. Hence, $\mathbf{t}_x$ and $\mathbf{t}_y$ can be written as

$$\mathbf{t}_x - i\mathbf{t}_y = \mathbf{C}\mathbf{f}'(z) \tag{5}$$

where the prime indicates the derivative with respect to the argument.

Using (5), the resultant force and the sum of the normal component of the electric displacement along any arc AB can be represented as

$$\mathbf{T} = \int_A^B \mathbf{t}_x \,\mathrm{d}y - \mathbf{t}_y \,\mathrm{d}x = \mathbf{C}\,\mathrm{Im}[\mathbf{f}(z)]_A^B \tag{6}$$

where Im stands for the imaginary part.

Assuming the perfect bonding between the medium I and the medium II implies that

$$\mathbf{u}^{(1)}(z) = \mathbf{u}^{(2)}(z),\ \ \mathbf{T}^{(1)}(z) = \mathbf{T}^{(2)}(z) \qquad \text{along the interface} \tag{7}$$

Inserting (4) and (6) into (7) yields

$$\mathrm{Re}[\mathbf{f}^{(1)}(z)] = \mathrm{Re}[\mathbf{f}^{(2)}(z)], \mathbf{C}^{(1)}\,\mathrm{Im}[\mathbf{f}^{(1)}(z)] = \mathbf{C}^{(2)}\,\mathrm{Im}[\mathbf{f}^{(2)}(z)] \tag{8}$$

The following main task is to determine the complex potentials $\mathbf{f}^{(r)}(z)$ by using (8) and the discontinuity conditions of the elastic displacement and the electric potential for a piezoelectric screw dislocation.

The complex potentials $\mathbf{f}^{(r)}(z)$, $r=1,2$, which satisfy (8) can be written as

$$\mathbf{f}^{(1)}(z) = \mathbf{f}_s^{(1)}(z) + \mathbf{f}_p^{(1)}(z),\ \ \mathbf{f}^{(2)}(z) = \mathbf{f}_p^{(2)}(z) \tag{9}$$

where $\mathbf{f}_s^{(1)}(z)$ represents the function vector associated with the unperturbed fields which are related to the solution of the corresponding homogenous media subject to a piezoelectric screw dislocation and holomorphic in the entire domain except at z_0. The $\mathbf{f}_p^{(1)}(z)\,(\text{or}\,\mathbf{f}_p^{(2)}(z))$ is the holomorphic function vector corresponding to the perturbed fields of the domain S_1 (or the domain S_2). From Pak's results[8], we have

$$\mathbf{f}_s^{(1)}(z) = \mathbf{A}\ln(z - z_0),\ \ \mathbf{A} = \frac{1}{2\pi i}\mathbf{b} \tag{10}$$

Substituting (9) into (8) and then using standard analytic continuation arguments, we obtain

$$\mathbf{f}_p^{(1)}(z) = \mathbf{\Pi}\bar{\mathbf{f}}_p^{(1)}(z), \qquad z \in \mathrm{S}_1 \tag{11a}$$

$$\mathbf{f}_p^{(2)}(z) = (\mathbf{I} + \mathbf{\Pi})\mathbf{f}_p^{(1)}(z), \qquad z \in \mathrm{S}_2 \tag{11b}$$

where the overbar denotes the conjugate of a complex function, $\mathbf{I}$ is a 2×2 unit matrix and

$$\mathbf{\Pi} = (\mathbf{C}_1 + \mathbf{C}_2)^{-1}(\mathbf{C}_1 - \mathbf{C}_2) \tag{12}$$

With (11), the electroelastic fields are obtained in terms of (4) and (5) and the problem is thus solved.

Image force on a piezoelectric screw dislocation

One of the major interests in discussing the dislocation–interface interaction problems is the image force acting on a dislocation. According to the generalized Peach Koehler formula derived by Pak [8], the image forces on the piezoelectric screw dislocation are

$$F_x = \mathbf{b}^{\mathrm{T}}\mathbf{t}_{yp}^{(1)}(z_0),\quad F_y = -\mathbf{b}^{\mathrm{T}}\mathbf{t}_{xp}^{(1)}(z_0) \tag{13}$$

where $\mathbf{t}_{jp}^{(1)}(j = x, y)$ are the perturbed stresses and electric displacements at z_0. Using (2), (11a) and (13), one obtains

$$F_x = 0,\quad F_y = -\frac{1}{4\pi y_0}\mathbf{b}^{\mathrm{T}}\mathbf{C}^{(1)}\mathbf{\Pi}\,\mathbf{b} \tag{14}$$

(13) shows that a dislocation can not move along the x-direction. If F_y<0, then the medium I will attract the dislocation; Otherwise, it will repel the dislocation. Letting $\mathbf{C}_1$=0, $(14)_2$ reduces to

$$F_y = -\frac{1}{4\pi y_0}(C_{44}^{(1)}b_3^2 + 2e_{15}^{(1)}b_3 b_\varphi - \varepsilon_{11}^{(1)}b_\varphi^2) \tag{15}$$

which agrees with the previous result of Pak[8] for the piezoelectric screw dislocation interacting with the free surface of the piezoelectric half-plane.

Numerical results and discussions

In this section, we will use $(14)_2$ to investigate the influence of the piezoelectric behavior and the material moduli ratios on the image force when $b_\varphi = 0$. The strength of piezoelectricity is characterized by the electrical-mechanical coupling factor $k = \sqrt{e_{15}^2/(C_{44}\varepsilon_{11})}$ 。The image force F_y is normalized as

$$F_{img} = \frac{4\pi y_0}{C_{44}^{(1)}b_3^2}F_y \tag{16}$$

Figure 2 shows the variations of the normalized image force F_{img} vs the shear moduli ratio $C_{44}^{(2)}/C_{44}^{(1)}$ for the piezoelectric bimaterial consisting of the piezoelectric medium I and the purely elastic medium II. The material constants of the medium I are listed in Table 1. k = 0 denotes that a medium is purely elastic. It is observed from Fig.2 that: (1) the stronger the electrical-mechanical coupling effect, the larger the image force F_{img}; (2) *the interface may also repel the dislocation located in the material with a higher shear modulus*, which is different from the conclusion obtained by Head[1] for the purely elastic case.

Table 1 Material constants for the medium I

materials	C_{44}($\times10^{10}$N/m^2)	e_{15}(C/m^2)	ε_{11}($\times10^{-10}$C/Vm)	k
PZT−6B	2.71	4.6	36	0.271
PZT- 5H	3.53	17	151	0.542

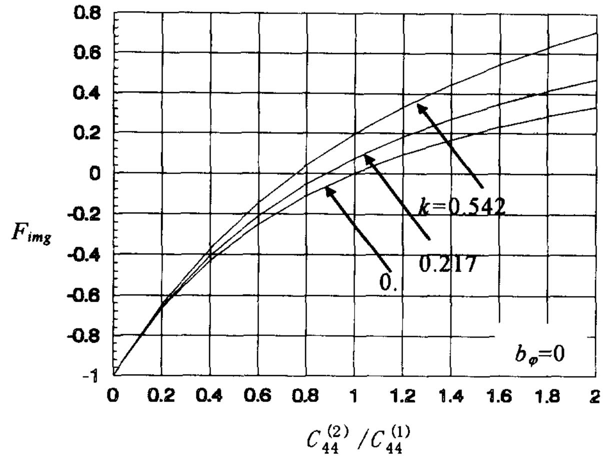

Fig.2 Variation of the normalized image force F_{img} vs shear moduli $C_{44}^{(2)}/C_{44}^{(1)}$

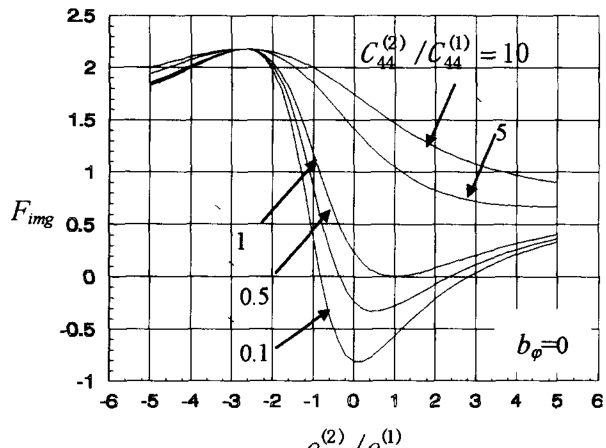

Fig.3 Variations of the normalized image force F_{img} vs the piezoelectric moduli ratio $e_{15}^{(2)}/e_{15}^{(1)}$

Figure 3 shows that the variations of the normalized image force F_{img} vs the piezoelectric moduli ratio $e_{15}^{(2)}/e_{15}^{(1)}$ for the different shear moduli ratios $C_{44}^{(2)}/C_{44}^{(1)}$ and $\varepsilon_{11}^{(2)}/\varepsilon_{11}^{(1)}=1$. The medium I is taken as PZT-5H piezoelectric ceramic. That $e_{15}^{(2)}/e_{15}^{(1)}<0$ represents that the poling directions of the two materials are opposite. Figure 3 indicates that: (1) the interface always repels the dislocation in the material with a lower shear modulus; (2) when the medium II is softer the medium I ($C_{44}^{(2)}/C_{44}^{(1)}<1$), whether the interface repels or attracts the dislocation depends on the piezoelectric moduli ratio $e_{15}^{(2)}/e_{15}^{(1)}$ of the two materials.

Conclusions

The electroelastic interaction between a screw dislocation and an interface of a piezoelectric bimaterial has been studied. The main conclusions are:

(1) The interface always repels the dislocation in the lower shear modulus, this being identical to the result for the purely elastic case.

(2) The interface may also repel the dislocation in the higher shear modulus due the presence of electro-mechanical coupling effect, which is different from the result for the purely elastic case.

Acknowledgements: The first author would like to appreciate referees' valuable comments and Professor B. A. Boley's suggestions on revising the manuscript.

References

1. A.K. Head, *Phil. Mag.* **44**, 92(1953)
2. J. Dundurs and G.P. Sendeckyj, *J. Appl. Phys.* **36**, 3353(1965)
3. D.M. Barnett and J. Lothe, *J. Phys.* **4**, 1618(1974)
4. T.C.T.Ting and D.M. Barnett, *Int. J. Solids Struct.* **30**, 313(1993)
5. H. fan and Z.M. Xiao, *Mech. Mater.* **23**, 21(1996)
6. J.D.Eshelby, *Phil. Trans. Roy. Soc.*, **A244**, 87(1951)
7. Y.E.Pak, *J. Appl. Mech.* **57**, 863(1990)
8. D.M.Barnett and J.Lothe, *Phys. Stat. Sol. (b)*. **67, 105(**1975**)**
9. Z. Suo, *Proc. R. Soc. Lond.* **A427**, 331(1990)