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INVESTIGATION OF A GRIFFITH CRACK SUBJECT TO UNIFORM TENSION USING THE NON-LOCAL THEORY BY A NEW METHOD *

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Abstract: *Field equations of the non-local elasticity are solved to determine the state of stress in a plate with a Griffith crack subject to uniform tension. Then a set of dual-integral equations is solved using a new method, namely Schmidt's method. This method is more exact and more reasonable than Eringen's one for solving this kind of problem. Contrary to the solution of classical elasticity, it is found that no stress singularity is present at the crack tip. The significance of this result is that the fracture criteria are unified at both the macroscopic and the microscopic scales. The finite hoop stress at the crack tip depends on the crack length.*

Key words: non-local theory; Schmidt's method; dual-integral equation

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Introduction

In several previous papers [1, 2, 3], Eringen discussed the state of stress near the tip of a sharp line crack in an elastic plate subject to uniform tension, shear and anti-plane shear. The field equations employed in the solution of these problems are those of the theory of non-local elasticity. The solutions obtained did not contain any stress singularity, thus resolving a fundamental problem persisted over many years. In the papers [4, 5], Zhou had discussed the scattering of the harmonic waves by two collinear cracks and by a Griffith crack using the non-local theory, respectively. And in papers [6, 7], they discussed the propagation of Love wave and the wave propagation in elastic plate by use of non-local theory, respectively. This enables us to employ the maximum stress hypothesis to deal with fracture problems in a natural way, and also the non-local elasticity has a big potential to understand the behavior of composite materials. However, Eringen's solution is not exact[1, 2, 3]. The stress solution of Eringen's [1] has oscillations near the crack tip for one dimensional problem. For a large lattice parameter, the relative errors of Eringen's [1] solution will become large. For this reason, the iterative technique used by Eringen[1] was not advantageous for solving this kind of problem. The methods used by Eringen[2, 3] were not also advantageous for solving dual-integral equation, because the kernel of the second kind Fredholm integral equation in Eringen's papers [2, 3] is divergent.

In this paper, the same problem which was treated by Eringen[1] is resolved using a some-

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what different approach. Fourier transform is applied and a mixed boundary value problem is reduced to a set of dual-integral equations. In solving the dual-integral equations, the crack surface displacement is expanded in a series using Jacobi's polynomials and Schmidt's method is used. This process is quite different from that adopted in Eringen's papers [1, 2, 3]. This method can overcome difficulties that occur in Eringen's papers [1, 2, 3]. The solution in this paper is more accurate and more reasonable than Eringen's. As expected, it does not contain the stress singularity at the crack tip, thus clearly indicating the physical nature of the problem, namely, in the vicinity of a geometrical discontinuity in the body, the non-local intermolecular forces are dominant. For such problems, therefore, one must resort to theories incorporating non-local effects, at least in the neighborhood of the discontinuity. The stress along the crack line depends on the crack length.

1 Basic Equations of Non-Local Elasticity

Basic equations of linear, homogeneous, isotropic, non-local elastic solids, with vanishing body force are[2]

$$\tau_{kl,\,k} = 0, \tag{1}$$

$$\tau_{kl} = \int_V [\lambda'(|X'-X|)e_{rr}(X')\,\delta_{kl} + 2\mu'(|X'-X|)e_{kl}(X')]\,dV', \tag{2}$$

$$e_{kl} = \frac{1}{2}(u_{k,\,l} + u_{l,\,k}). \tag{3}$$

Where the only difference from classical elasticity is in the stress constitutive equation (2) in which the stress $\tau_{kl}(X)$ at a point X depends on the strains $e_{kl}(X')$, at all points of the body. For homogeneous and isotropic solids there exist only two material parameters, $\lambda'(|X'-X|)$ and $\mu'(|X'-X|)$ which are functions of the distance $|X'-X|$. The integral in equation (2) is over the volume V of the body enclosed within a surface ∂V.

In this paper we employ Cartesian coordinates x_k with the usual convention that a free index takes the values (1, 2, 3), and repeated indices are summed over the range (1, 2, 3). Indices following a comma represent the partial differentiation with respect to the coordinate, e. g.

$$u_{k,\,l} = \partial u_k / \partial x_j.$$

In the papers [8, 9], the form of $\lambda'(|X'-X|)$ and $\mu'(|X'-X|)$ is obtained for which the dispersion curves of plane elastic waves coincide with those known in lattice dynamics. Among several possible curves the following has been found to be very useful

$$(\lambda',\ \mu') = (\lambda,\ \mu)\alpha(|X'-X|), \tag{4}$$

$$\alpha(|X'-X|) = \alpha_0 \exp[-(\beta/a)^2(X'-X)(X'-X)], \tag{5}$$

where β is a constant, α is the lattice parameter. λ and μ are the Lame constants of classical elasticity. α_0 is determined by the normalization condition

$$\int_V \alpha(|X'-X|)\,dV(X') = 1. \tag{6}$$

In the present work we employ the non-local elastic moduli given by equations (4) and (5). Substituting equation (5) into equation (6) we obtain, in two dimensional space,

$$\alpha_0 = \frac{1}{\pi}(\beta/a)^2. \tag{7}$$

Substituting equations (4) and (5) into equation (2) yields

$$\tau_{kl}(X) = \int_V \alpha(|X' - X|)\sigma_{kl}(X')\mathrm{d}V(X'), \tag{8}$$

where

$$\sigma_{ij}(X') = \lambda e_{rr}(X')\delta_{ij} + 2\mu e_{ij}(X') = \lambda u_{r,r}(X')\delta_{ij} + \mu[u_{i,j}(X') + u_{j,i}(X')]. \tag{9}$$

The expression (9) is the classical Hook's law. Substituting equation (9) into equation (1) and using Green-Gauss theorem, it can be obtained:

$$\int_V \alpha(|X' - X|)\sigma_{kl,k}(X')\mathrm{d}V(X') - \int_{\partial V} \alpha(|X' - X|)\sigma_{kl}(X')\mathrm{d}a_k(X') = 0. \tag{10}$$

Here the surface integral may be dropped since the displacement field vanishes at infinity.

2 The Crack Model

We consider an elastic plate in the ($x_1 = x$, $x_2 = y$) plane weakened by a line crack of length $2l$ along the x-axis. The plate is subjected to uniform stress $\tau_{yy} = \tau_0$ along the surface of the crack, Fig 1. In the plane strain case, equation (10) takes the form

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \alpha(|x' - x|, |y' - y|)\sigma_{kj,k}(x', y')\mathrm{d}x'\mathrm{d}y' -$$

$$2\mu\int_{-l}^{l} \alpha(|x' - x|, 0)[\![e_{2j}(x', 0)]\!]\mathrm{d}x' = 0, \tag{11}$$

where the boldface bracket indicates the jump across the crack line.

Using the method of the paper [1], we can obtain

$$[\![e_{2j}(x, 0)]\!] = 0, \qquad (j = 1, 2, \text{ for all } x). \tag{12}$$

The Fourier transform of equation (11) with respect to x' gives

$$-\mathrm{i}s\bar{\sigma}_{1j} + \mathrm{d}\bar{\sigma}_{2j}/\mathrm{d}y = 0, \qquad j = 1, 2, \tag{13}$$

where a superposed bar indicates the Fourier transform, e. g.,

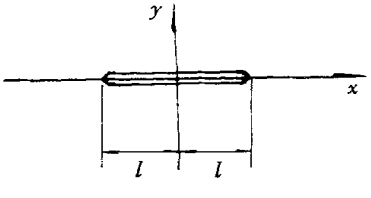

Fig. 1 Line crack subject to uniform tension

$$\bar{f}(s) = \int_{-\infty}^{\infty} f(x)\mathrm{e}^{-\mathrm{i}sx}\mathrm{d}x. \tag{14}$$

As discussed in the reference [1] the boundary conditions at $y = 0$ are

$$\tau_{yx}(x, 0) = 0, \quad \tau_{yy}(x, 0) = \tau_0, \qquad |x| < l, \tag{15}$$

$$\tau_{yx}(x, 0) = 0, \quad v(x, 0) = 0, \qquad |x| > 1. \tag{16}$$

In addition we must have

$$u = v = 0, \qquad \text{as } (x^2 + y^2)^{1/2} \to \infty. \tag{17}$$

Consequently we must obtain the solution of equation (13) subject to the boundary condi-

tions (15 ~ 17). Equation (13) is none other than the Fourier transforms of the Navier's equations in two dimension case, namely

$$\mu\bar{u}_{,yy} - (\lambda+2\mu)s^2\bar{u} - is(\lambda+\mu)\bar{v}_{,y} = 0, \tag{18}$$

$$-is(\lambda+\mu)\bar{u}_{,y} + (\lambda+2\mu)\bar{v}_{,yy} - s^2\mu\bar{v} = 0. \tag{19}$$

Because of symmetry, it suffices to consider the problem in the first quadrant only. The general solution of this set ($y \geqslant 0$) satisfying (17) are:

$$u = \frac{2}{\pi}\int_0^\infty s^{-1}\left[sA(s) + \left(sy - \frac{\lambda+3\mu}{\lambda+\mu}\right)B(s)\right]\exp(-sy)\sin(xs)\,ds, \tag{20}$$

$$v = \frac{2}{\pi}\int_0^\infty [A(s) + yB(s)]\exp(-sy)\cos(sx)\,ds, \tag{21}$$

where $A(s)$ and $B(s)$ are to be determined from the boundary conditions (15) and (16). Using equation (9) we obtain ($y \geqslant 0$)

$$\sigma_{yx}(x, y) = \frac{4\mu}{\pi}\int_0^\infty\left[-sA(s) + \left(\frac{\lambda+2\mu}{\lambda+\mu} - sy\right)B(s)\right]\exp(-sy)\sin(sx)\,ds. \tag{22}$$

According to equations (15) and (16), this must vanish at $y = 0$. Hence

$$B(s) = \frac{\lambda+\mu}{\lambda+2\mu}sA(s). \tag{23}$$

Noting that $A(-s) = A(s)$, on account of symmetry $v(x, y) = v(-x, y)$, the displacement field may be put into the from

$$u = \frac{-2(\lambda+\mu)}{\pi(\lambda+2\mu)}\int_0^\infty A(s)\left(\frac{\mu}{\lambda+\mu} - sy\right)\exp(-sy)\sin(sx)\,ds, \tag{24}$$

$$v = \frac{2(\lambda+\mu)}{\pi(\lambda+2\mu)}\int_0^\infty A(s)\left(\frac{\lambda+2\mu}{\lambda+\mu} + sy\right)\exp(-sy)\cos(sx)\,ds. \tag{25}$$

For the σ_{kl}, through equations (9) and (23) we obtain ($y \geqslant 0$)

$$\sigma_{yy}(x, y) = -\frac{4(\lambda+\mu)\mu}{\pi(\lambda+2\mu)}\int_0^\infty sA(s)(1+sy)\exp(-sy)\cos(sx)\,ds. \tag{26}$$

The stress field, according to equation (8), is then given by

$$\tau_{yy}(x, y) = \int_0^\infty dy'\int_{-\infty}^\infty \sigma_{yy}(x', y')[\alpha(|x'-x|, |y'-y|) + \alpha(|x'-x|, |y'+y|)]\,dx'. \tag{27}$$

Substituting for α from equation (5), according to the reference [2] and the boundary conditions, it can be obtained

$$\int_0^\infty sA(s)k(\varepsilon s)\cos(sx)\,ds = -\frac{\pi\tau_0(\lambda+2\mu)}{4\mu(\lambda+\mu)}, \qquad |x| < l, \tag{28}$$

$$\int_0^\infty A(s)\cos(sx)\,ds = 0, \qquad l \leqslant |x|, \tag{29}$$

$$k(\xi) = [1 - \Phi(\xi)][1 + 2\xi^2] - \frac{\xi}{\sqrt{\pi}}\exp(-\xi^2), \tag{30}$$

where $$\varepsilon = \frac{a}{2\beta},\quad \Phi(z) = \frac{2}{\sqrt{\pi}}\int_0^z \exp(-t^2)\,dt. \tag{31}$$

The only difference between the classical and non-local equations is in the introduction of the function $k(\varepsilon s)$. It is logical to utilize the classical solution to convert the system (28) and (29) to an integral equation of the second kind which is generally better behaved. For $\alpha = 0$, then $k(\varepsilon s) = 1$, then equations (28) and (29) reduce to a set of dual integral equations for the same problem in classical elasticity. To determine the unknown function $A(s)$, we must solve the dual-integral equations (28) and (29).

3 Solution of the Dual-Integral Equation

Of course, the dual-integral equations (28) and (29) can be considered to be a single integral equation of the first kind with a discontinuous kernel. It is well-known in the literature that integral equations of the first kind are generally ill-posed in the sense of Hadamard, i. e. small perturbations of the data can yield arbitrarily large changes in the solution. The iterative technique used by Eringen [1] was not advantageous for solving this kind of problem, because the relative errors of Eringen's [1] solution is large for a large lattice parameter. This makes the numerical solution of such equations quite difficult. For overcoming the difficult, the Schmidt method[10] is used to solve the dual integral equations (28) and (29). The displacement v can be represented by the following series:

$$v(x,0) = \sum_{n=1}^{\infty} a_n P_{2n-2}^{(1/2,\,1/2)}\left(\frac{x}{l}\right)\left(1-\frac{x^2}{l^2}\right)^{1/2}, \qquad \text{for } 0 \leqslant |x| \leqslant l, \tag{32}$$

$$v(x,0) = 0, \qquad \text{for } l \leqslant |x|, \tag{33}$$

where a_n are unknown coefficients to be determined and $P_n^{(1/2,\,1/2)}(x)$ is a Jacobi polynomial[11]. The Fourier cosine transform for equation (32) is[12]

$$A(s) = \bar{v}(s,0) = \sum_{n=1}^{\infty} a_n B_n J_{2n-1}(ls)s^{-1}, \tag{34}$$

where $$B_n = 2\sqrt{\pi}(-1)^{n-1}\frac{\Gamma(2n-1/2)}{(2n-2)!}, \tag{35}$$

and $\Gamma(x)$ and $J_n(x)$ are the Gamma and Bessel functions, respectively.

Substituting equation (34) into equations (28) and (29), respectively, equation (29) has been automatically satisfied by using the Fourier transform. Then the remaining equation can be reduced to the form for $x \leqslant l$

$$\sum_{n=1}^{\infty} a_n B_n \int_0^{\infty} k(\varepsilon s) J_{2n-1}(sl)\cos(sx)\,ds = -\frac{\pi\tau_0(\lambda+2\mu)}{4\mu(\lambda+\mu)}. \tag{36}$$

For a large $\xi = \varepsilon s$, the integrands of the equation (36) almost all decrease exponentially. So the semi-infinite integral in equation (36) can be evaluated numerically by Filon's method[13]. Thus equation (28) can be solved for coefficients a_n by the Schmidt method[10]. For brevity, we have rewritten equation (36) as

$$\sum_{n=1}^{\infty} a_n E_n(x) = U(x), \qquad 0 \leqslant x \leqslant l, \tag{37}$$

where $E_n(x)$ and $U(x)$ are known functions and coefficients a_n are unknown and will be deter-

mined. A set of functions $P_n(x)$, which satisfy the orthogonality condition

$$\int_0^l P_m(x)P_n(x)\mathrm{d}x = N_n\delta_{mn},\quad N_n = \int_0^l P_n^2(x)\mathrm{d}x \tag{38}$$

can be constructed from the function, $E_n(x)$. such that

$$P_n(x) = \sum_{i=1}^{n}\frac{M_{in}}{M_{nn}}E_i(x), \tag{39}$$

where M_{in} is the cofactor of the element d_{in} of $\boldsymbol{D}_n$, which is defined as

$$\boldsymbol{D}_n = \begin{bmatrix} d_{11} & d_{12} & d_{13} & \cdots & d_{1n} \\ d_{21} & d_{22} & d_{23} & \cdots & d_{2n} \\ d_{31} & d_{32} & d_{33} & \cdots & d_{3n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ d_{n1} & d_{n2} & d_{n3} & \cdots & d_{nn} \end{bmatrix},\quad d_{ij} = \int_0^l E_i(x)E_j(x)\mathrm{d}x. \tag{40}$$

Using equations (37) ~ (39), we obtain

$$a_n = \sum_{j=n}^{\infty} q_j\frac{M_{nj}}{M_{jj}}, \tag{41}$$

with

$$q_j = \frac{1}{N_j}\int_0^l U(x)P_j(x)\mathrm{d}x. \tag{42}$$

4 Numerical Calculations and Discussion

For a check of accuracy, the values of $\sum_{n=1}^{10} a_nE_n(x)$ and $U(x)$ are given in Table 1 in the cases of $a/2\beta l = 0.000\,5$. In Table 2, the values of the coefficients a_n are given for $a/2\beta l = 0.000\,5$.

Table 1 Values of $\sum_{n=1}^{10} a_nE_n(x)\Big/\frac{\pi\tau_0(\lambda+2\mu)}{4\mu(\lambda+\mu)}$ and $U(x)\Big/\frac{\pi\tau_0(\lambda+2\mu)}{4\mu(\lambda+\mu)}$ for $a/2\beta l = 0.000\,5$

x/l	$\sum_{n=1}^{10} a_nE_n(x)\Big/\frac{\pi\tau_0(\lambda+2\mu)}{4\mu(\lambda+\mu)}$	$U(x)\Big/\frac{\pi\tau_0(\lambda+2\mu)}{4\mu(\lambda+\mu)}$
0.55	−0.100 309 E+01	−1.0
0.60	−0.100 531 E+01	−1.0
0.75	−0.995 341 E+00	−1.0
0.80	−0.100 439 E+01	−1.0
0.90	−0.996 281 E+00	−1.0
0.95	−0.102 530 E+01	−1.0
0.96	−0.102 805 E+01	−1.0
0.97	−0.102 363 E+01	−1.0
0.98	−0.998 501 E+00	−1.0
0.99	−0.869 906 E+00	−1.0

Table 2 Values of $a_n\left|\frac{\pi\tau_0(\lambda+2\mu)}{4\mu(\lambda+\mu)}\right.$ for $a/2\beta l=0.0005$

n	$a_n\left\|\frac{\pi\tau_0(\lambda+2\mu)}{4\mu(\lambda+\mu)}\right.$	n	$a_n\left\|\frac{\pi\tau_0(\lambda+2\mu)}{4\mu(\lambda+\mu)}\right.$
1	−0.318 698 E+00	6	−0.132 851 E−03
2	−0.127 109 E−01	7	−0.570 583 E−04
3	0.708 155 E−02	8	−0.981 545 E−04
4	0.174 376 E−02	9	−0.106 541 E−04
5	0.127 016 E−02	10	−0.582 841 E−05

From the above results and references [14], [15], it can be seen that the Schmidt method is performed satisfactorily if the first ten terms of the infinite series of equation (37) are obtained. The behavior of the stress stays steady with the increasing number of terms in equation (32). When coefficients a_n are known, the entire stress field is obtainable. However, in fracture mechanics, it is of importance to determine stress τ_{yy} along the crack line. τ_{yy} at $y=0$ is given as follows:

$$\tau_{yy}=-\frac{4\mu(\lambda+\mu)}{\pi(\lambda+2\mu)}\sum_{n=1}^{\infty}a_nB_n\int_0^{\infty}k(\varepsilon s)J_{2n-1}(sl)\cos(sx)\mathrm{d}s. \tag{43}$$

For $\varepsilon=0$ at $x=l$, it has the classical stress singularity. However, so long as $\varepsilon\neq0$, equation (43) gives a finite stress all along $y=0$. At $0<x<l$, τ_{yy}/τ_0 is very close to unity, and for $x>l$, τ_{yy}/τ_0 possesses finite values diminishing from a finite value at $x=l$ to zero at $x=\infty$. Since $\varepsilon/l>1/100$ represents a crack length of less than 10^{-6} cm, and for such submicroscopic sizes other serious questions arise regarding the interatomic arrangements and force laws, we do not pursue solutions valid at such small crack sizes. The stress is computed numerically for Poisson's ratio $\nu=0.29$. The semi-infinite numerical integrals, which occur, are evaluated easily by Filon and Simpson's methods[13] because the rapid diminution of the integrands. The results are plotted in Figs. 2~7.

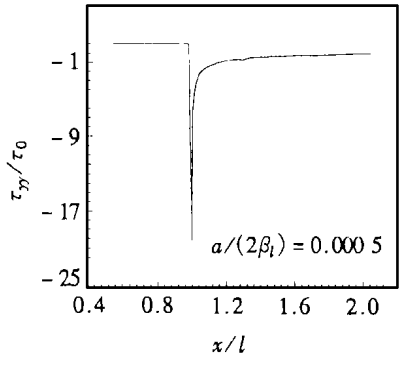

Fig. 2 Stress along the crack direction

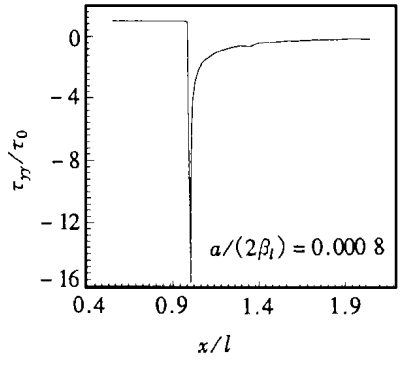

Fig. 3 Stress along the crack direction

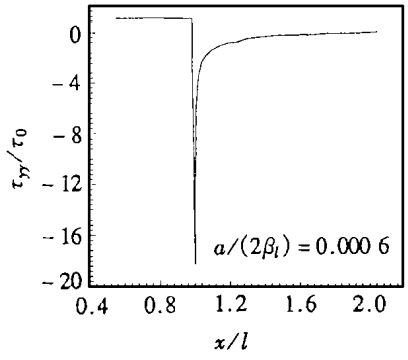

Fig. 4 Stress along the crack direction

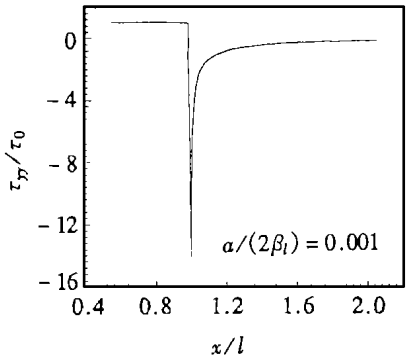

Fig. 5 Stress along the crack direction

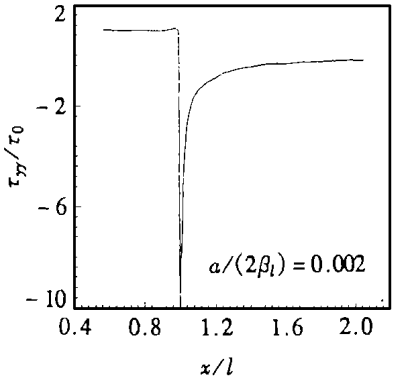

Fig. 6 Stress along the crack direction

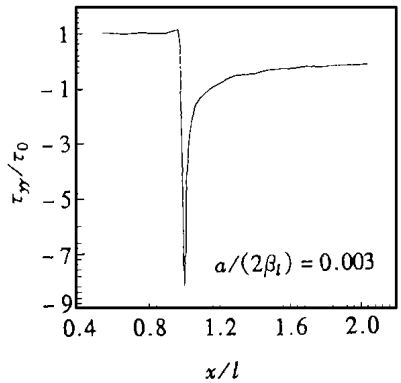

Fig. 7 Stress along the crack direction

The following observations are made:

(i) The method used in this paper can overcome difficulties that occur in Eringen's papers [1, 2, 3,]. The results are more accurate than Eringen's ones. The method is more reasonable than Eringen's ones.

(ii) The maximum stress does not occur at the crack tip, but slightly away from it. This phenomenon has been thoroughly substantiated by Eringen[16]. The maximum stress is finite. The distance between the crack tip and the maximum stress point is very small. Contrary to the classical elasticity solution, it is found that no stress singularity is present at the crack tip, and also the present results converge to the classical ones for positions when far away from the crack tip.

(iii) The stress at the crack tip becomes infinite when the atomic distance $a \to 0$. This is the classical continuum limit of square root singularity.

(iv) If $\alpha/\beta =$ constant, viz., the atomic distance does not change, the value of the stress concentrations (at the crack tip) becomes higher with the increase of the crack length. Note the fact that experiments indicate that materials with smaller cracks are more resistant to fracture than those with larger cracks.

(V) The significance of this result is that the fracture criteria are unified at both the macro-

scopic and microscopic scales.

(Ⅵ) The stress concentration occurs at the crack tip, and this is given by

$$\tau_{yy}(l, 0)/\tau_0 = c_2\sqrt{2\beta l/a}, \tag{44}$$

where c_2 converges to $c_2 \approx 0.315$.

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