Effective behavior of piezoelectric composites

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In this paper, general relations between the overall properties of piezoelectric composites and the properties of their constituents are derived. Based on the solution for an ellipsoidal inclusion in a piezoelectric material developed by Wang (*Int. J. Solids and Structures*, **29**,293,1992), it is found that the coupled elastic and electric field inside a spheroidal inclusion in a transversely isotropic, piezoelectric matrix can be expressed in terms of a system of the linear algebraic equations which contains only some simple integrals. These internal fields are then used to obtain the effective constants of a piezoelectric composite.

INTRODUCTION

Piezoelectric composites can be used for optimizing the coupled field behavior of some technological devices. Since the late seventies, they have been used in the manufacturing of ultrasonic transducers. Theoretical work concerning the overall behavior of piezoelectric composites is rather scarce, especially in comparison with numerous results and micromechanics models which exist today on the uncoupled mechanical behavior. One reason is because piezoelectric media are intrinsically anisotropic and the electric field and elastic field are coupled in such materials, these facts make the mathematical analysis is rather difficult. The analysis is even more difficult when an inclusion is introduced. The existing works are some simple micromechanical modelings which have allowed the technological implementation of such materials. Recently, Schulgasser (1992), Benveniste and Dvorak (1992), and Wang (1992) have done some works which are based on the continuum analysis. Dunn and Taya (1993) used several popular methods such as the self-consistent, Mori-Tanaka and differential schemes which has been used in predicting the uncoupled behavior of composite to predict the effective electroelstic moduli of piezoelectric composites. To predict the effective properties of such composites, one needs a solution of a single inclusion in an infinite piezoelectric solid. Dunn and Taya's prediction was based on a solution which contains some surface integrals over the unit sphere. Thus such a solution is not very easy to use in engineering. By considering that the general piezoelectric solid is transversely isotropic, the present author derived a solution which contains only some single integrals, and seems to be the final analytical result for such piezoelectric materials. Based on this explicit solution, the effective constants of composites which are modeled as the material system consisting of transversely isotropic, piezoelectric matrix and transversely isotropic, piezoelectric spheroidal inclusions whose symmetric axes parallel with the poling axis are derived in this paper by using the Mori-Tanaka approach.

THE COUPLED ELASTIC AND ELECTRIC FIELD INSIDE A SPHEROIDAL INCLUSION

Consider an infinite piezoelectric body with the elastic moduli tensor $\mathbf{C}^0$, piezoelectric tensor $\mathbf{e}^0$, and dielectric permittivity α^0 in which there is an ellipsoidal inclusion with constants $\mathbf{c}$, $\mathbf{e}$ and α

$$\frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2} + \frac{x_3^2}{a_3^2} = 1 \quad (1)$$

If the material is subjected to the uniform elastic field ε^0 and electric field $\mathbf{E}^0$, the elastic field ε^I and the electric field $\mathbf{E}^I$ inside the inclusion remain uniform, and are given by the following equations

$$\varepsilon_{ij}^I = B_{ijkl}^1 \varepsilon_{kl}^0 + B_{ijk}^2 E_k^0 \quad (2)$$

$$E_i^I = B_{ijk}^3 \varepsilon_{jk}^0 + B_{ik}^4 E_k^0 \quad (3)$$

where the tensors $\mathbf{B}^1$, $\mathbf{B}^2, \mathbf{B}^3$ and $\mathbf{B}^4$ are constants which depend on the inclusion shape and material constants. If the inclusion is of spheroidal shape, i. e. $a_1 = a_2 = a$, and the constant tensors can be simplified. For convenience, we express the relations (2) and (3) in matrix form for spheroidal inclusion.

$$\begin{bmatrix} \varepsilon^I_{11} \\ \varepsilon^I_{22} \\ \varepsilon^I_{33} \\ \varepsilon^I_{23} \\ \varepsilon^I_{13} \\ \varepsilon^I_{12} \end{bmatrix} = \begin{bmatrix} B^1_{11} & B^1_{12} & B^1_{13} & 0 & 0 & 0 \\ B^1_{12} & B^1_{11} & B^1_{13} & 0 & 0 & 0 \\ B^1_{31} & B^1_{31} & B^1_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & B^1_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & B^1_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & B^1_{66} \end{bmatrix} \times \begin{bmatrix} \varepsilon^0_{11} \\ \varepsilon^0_{22} \\ \varepsilon^0_{33} \\ \varepsilon^0_{23} \\ \varepsilon^0_{13} \\ \varepsilon^0_{12} \end{bmatrix} + \begin{bmatrix} 0 & 0 & B^2_{13} \\ 0 & 0 & B^2_{13} \\ 0 & 0 & B^2_{33} \\ 0 & B^2_{42} & 0 \\ B^2_{42} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E^0_1 \\ E^0_2 \\ E^0_3 \end{bmatrix} \tag{4}$$

$$\begin{bmatrix} E^I_1 \\ E^I_2 \\ E^I_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & B^3_{15} & 0 \\ 0 & 0 & 0 & B^3_{15} & 0 & 0 \\ B^3_{31} & B^3_{31} & B^3_{33} & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} \varepsilon^0_{11} \\ \varepsilon^0_{22} \\ \varepsilon^0_{33} \\ \varepsilon^0_{23} \\ \varepsilon^0_{13} \\ \varepsilon^0_{12} \end{bmatrix} + \begin{bmatrix} B^4_{11} & 0 & 0 \\ 0 & B^4_{11} & 0 \\ 0 & 0 & B^4_{33} \end{bmatrix} \begin{bmatrix} E^0_1 \\ E^0_2 \\ E^0_3 \end{bmatrix} \tag{5}$$

where

$$\begin{bmatrix} B^1_{11} & B^1_{12} & B^1_{13} \\ B^1_{12} & B^1_{11} & B^1_{13} \\ B^1_{31} & B^1_{31} & B^1_{33} \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{12} & R_{11} & R_{13} \\ R_{31} & R_{31} & R_{33} \end{bmatrix}^{-1} \tag{6}$$

and

$$R_{11} = 1 + \frac{1}{4\pi}(G^1_{11}c^1_{11} + G^1_{12}c^1_{12} + G^1_{13}c^1_{13} + G^2_{31}e^1_{31}) - \frac{\Delta}{16\pi^2}(G^1_{11}e^1_{31} + G^1_{12}e^1_{31} + G^1_{13}e^1_{33} - G^2_{31}\alpha^1_{33}) \times (G^2_{31}c^1_{11} + G^2_{31}c^1_{12} + G^2_{33}c^1_{33} + G^3_{33}e^1_{31}) \tag{7}$$

$$R_{33} = 1 + \frac{1}{4\pi}(G^1_{13}c^1_{13} + G^1_{13}c^1_{13} + G^1_{33}c^1_{33} + G^2_{33}e^1_{33}) - \frac{\Delta}{16\pi^2}(G^1_{13}e^1_{31} + G^1_{13}e^1_{31} + G^1_{33}e^1_{33} - G^2_{33}\alpha^1_{33}) \times (G^2_{31}c^1_{13} + G^2_{31}c^1_{13} + G^2_{33}c^1_{33} + G^3_{33}e^1_{33}) \tag{8}$$

$$R_{12} = \frac{1}{4\pi}(G^1_{11}c^1_{12} + G^1_{12}c^1_{11} + G^1_{13}c^1_{13} + G^2_{31}e^1_{31}) - \frac{\Delta}{16\pi^2}(G^1_{11}e^1_{31} + G^1_{12}e^1_{31} + G^1_{13}e^1_{33} - G^2_{31}\alpha^1_{33}) \times (G^2_{31}c^1_{11} + G^2_{31}c^1_{12} + G^2_{33}c^1_{33} + G^3_{33}e^1_{31}) \tag{9}$$

$$R_{13} = \frac{1}{4\pi}(G^1_{11}c^1_{13} + G^1_{12}c^1_{13} + G^1_{13}c^1_{33} + G^2_{31}e^1_{33}) - \frac{\Delta}{16\pi^2}(G^1_{11}e^1_{31} + G^1_{12}e^1_{31} + G^1_{13}e^1_{33} - G^2_{31}\alpha^1_{33}) \times (G^2_{31}c^1_{13} + G^2_{31}c^1_{13} + G^2_{33}c^1_{33} + G^3_{33}e^1_{33}) \tag{10}$$

$$R_{31} = \frac{1}{4\pi}(G^1_{13}c^1_{11} + G^1_{13}c^1_{12} + G^1_{33}c^1_{13} + G^2_{33}e^1_{31}) - \frac{\Delta}{16\pi^2}(G^1_{13}e^1_{31} + G^1_{13}e^1_{31} + G^1_{33}e^1_{33} - G^2_{33}\alpha^1_{33}) \times (G^2_{31}c^1_{11} + G^2_{31}c^1_{12} + G^2_{33}c^1_{33} + G^3_{33}e^1_{31}) \tag{11}$$

$$\Delta = \left[1 + \frac{1}{4\pi}(2G^2_{31}e^1_{31} + G^2_{33}e^1_{33} - G^3_{33}\alpha^1_{33}\right]^{-1} \tag{12}$$

$$B^1_{44} = \left[1 + \frac{1}{\pi}G^1_{44}c^1_{44} + \frac{1}{2\pi}G^2_{24}e^1_{24}\right.$$

$$- \frac{1}{2\pi^2} (G^1_{44}e^1_{24} - \frac{1}{2} G^2_{24}\alpha^1_{11})$$
$$\times(1 + \frac{1}{2\pi} G^2_{24}e^1_{24} - \frac{1}{4\pi} G^3_{11}\alpha^1_{11})^{-1}$$
$$\times(G^2_{24}c^1_{44} + \frac{1}{2} G^3_{11}e^1_{24})\Big]^{-1} \quad (13)$$

$$B^1_{66} = (1 + \frac{1}{\pi} G^1_{66}c^1_{66})^{-1} \quad (14)$$

$$B^2_{13} = \frac{\Delta}{4\pi} (B^1_{11} + B^1_{12})(G^1_{11}e^1_{31} + G^1_{12}e^1_{31} + G^1_{13}e^1_{33} - G^2_{31}\alpha^1_{33}) + \frac{\Delta}{4\pi} B^1_{13}(2G^1_{13}e^1_{31} + G^1_{33}e^1_{33} - G^2_{33}\alpha^1_{33}) \quad (15)$$

$$B^2_{33} = \frac{\Delta}{2\pi} B^1_{31} (G^1_{11}e^1_{31} + G^1_{12}e^1_{31} + G^1_{13}e^1_{33} - G^2_{31}\alpha^1_{33}) + \frac{\Delta}{4\pi} B^1_{33}(2G^1_{13}e^1_{31} + G^1_{33}e^1_{33} - G^2_{33}\alpha^1_{33}) \quad (16)$$

$$B^2_{42}=\frac{1}{2\pi} B^1_{44}(1+\frac{1}{2\pi} G^2_{24}e^1_{24}-\frac{1}{4\pi} G^3_{11}\alpha^1_{11})^{-1} (G^1_{44}e^1_{24} - \frac{1}{2} G^2_{24}\alpha^1_{11}) \quad (17)$$

$$B^3_{31} = \frac{\Delta}{4\pi} (B^1_{11} + B^1_{12})(G^2_{31}c^1_{11} + G^2_{31}c^1_{12} + G^1_{33}c^1_{13} + G^3_{33}e^1_{33}) + \frac{\Delta}{4\pi} B^1_{13}(2G^2_{31}c^1_{13} + G^2_{33}c^1_{33} + G^3_{33}e^1_{33}) \quad (18)$$

$$B^3_{33} = \frac{\Delta}{2\pi} B^1_{13}(G^2_{31}c^1_{11} + G^2_{31}c^1_{12} + G^1_{33}c^1_{13} + G^3_{33}e^1_{33}) + \frac{\Delta}{4\pi} B^1_{33}(2G^2_{31}c^1_{13} + G^2_{33}c^1_{33} + G^3_{33}e^1_{33}) \quad (19)$$

$$B^3_{15}=\frac{1}{\pi} B^1_{44}(1+\frac{1}{2\pi} G^2_{24}e^1_{24}-\frac{1}{4\pi} G^3_{11}\alpha^1_{11})^{-1} (G^2_{24}c^1_{44} + \frac{1}{2} G^3_{11}e^1_{24}) \quad (20)$$

$$B^4_{11} = (1 + \frac{1}{2\pi} G^2_{24}e^1_{24} - \frac{1}{4\pi} G^3_{11}\alpha^1_{11})^{-1} \times\Big[1 + \frac{1}{\pi} B^2_{42}(G^2_{24}c^1_{44} + \frac{1}{2} G^3_{11}e^1_{24})\Big] \quad (21)$$

$$B^4_{33} = \Delta + \frac{\Delta}{2\pi} B^2_{13}(G^2_{31}c^1_{11} + G^2_{31}c^1_{12} + G^1_{33}c^1_{13} + G^3_{33}e^1_{31}) + \frac{\Delta}{4\pi} B^2_{33}(2G^2_{31}c^1_{13} + G^2_{33}c^1_{33}) \quad (22)$$

and

$$c^1_{ij} = c_{ij} - c^0_{ij} \qquad , i , j = 1, 6 \quad (23)$$

$$e^1_{kj} = e_{kj} - e^0_{kj} \qquad , k = 1, 3 ; j = 1, 6 \quad (24)$$

$$\alpha^1_{kl} = \alpha_{kl} - \alpha^0_{kl} \qquad , k = 1, 3 ; l = 1, 3 \quad (25)$$

where c_{ij} , e_{kj}, and α_{kl} are the elastic, piezoelectric and dielectric constants of the inclusion in the form of matrix, and c^0_{ij} , e^0_{kj}, and α^0_{kl} are the material constants of the matrix. For transversely isotropic materials, they consist of the ten independent constants. (see, for example, Pak, 1990). The components of G^1_{ij} , G^2_{ij}, and G^3_{ij} , which are dependent on the aspect ratio of the spheroidal inclusion and the material behavior of the matrix, can be determined by some simple integrals, which are shown in Appendix A.

THE EFFECTIVE CONSTANTS OF PIEZOELECTRIC COMPOSITE WITH A DILUTE DISTRIBUTION ON INCLUSIONS

Definition. The effective elastic, piezoelectric and dielectric constants of piezoelectric composites, c^*_{ijkl} , e^*_{ijk}, and α^*_{ij} are defined by the following equations:

$$\langle\sigma_{ij}\rangle = c^*_{ijkl}\langle\varepsilon_{kl}\rangle - e^*_{mij}\langle E_m\rangle \quad (26)$$

$$\langle D_k\rangle = e^*_{kij}\langle\varepsilon_{ij}\rangle + \alpha^*_{kl}\langle E_l\rangle \quad (27)$$

where the symbol < > denotes the volume average.

From equation (26), it can be written that

$$\langle\sigma_{ij}\rangle = c^*_{ijkl}\langle\varepsilon_{kl}\rangle - e^*_{mij}\langle E_m\rangle = c^0_{ijkl}\langle\varepsilon_{kl}\rangle - e^0_{mij}\langle E_m\rangle + v_f(c^1_{ijkl}\langle\varepsilon^I_{kl}\rangle - e^1_{mij}\langle E^I_m\rangle) \quad (28)$$

where v_f is the volume fraction of inclusions. In the same manner, from equation (27), we obtain

$$<D_k> = e^*_{kij}<\varepsilon_{ij}> + \alpha^*_{kl}<E_l>$$
$$= e^0_{kij}<\varepsilon_{ij}> + \alpha^0_{kl}<E_l>$$
$$+ v_f(e^1_{kij}<\varepsilon^I_{ij}> + \alpha^1_{kl}<E^I_l>) \quad (29)$$

In the derivation of equations (28) and (29), we have used the constitutive relation of piezoelectric inclusion and matrix, which takes the same form as equations (26) and (27) with replacement of the correspondent material constants.

For the piezoelectric composite with a dilute distribution of inclusions, the interactions among inclusions can be neglected. If the material is subjected to constant external field such that

$$<\varepsilon_{ij}> = \varepsilon^0_{ij} \quad , \quad <E_k> = E^0_k \quad (30)$$

the average fields inside inclusions in equations (28) and (29) are assumed to be equal to the internal fields in a single inclusion in an infinite matrix, which can be determined by equations (2) and (3).

Substitution of equations (2) and (3) into equations (28) and (29) yields

$$c^*_{ijkl} = c^0_{ijkl} + v_f c^1_{ijmn}B^1_{mnkl} - v_f e^1_{mij}B^3_{mkl} \quad (31)$$

$$e^*_{mij} = e^0_{mij} - v_f c^1_{ijkl}B^2_{klm} + v_f e^1_{kij}B^4_{km} \quad (32)$$

$$\alpha^*_{kl} = \alpha^0_{kl} + v_f\alpha^1_{kj}B^4_{jl} + v_f e^1_{kij}B^2_{ijl} \quad (33)$$

Since the matrix and inclusion are all assumed to be transversely isotropic with the x_3-axis being the poling axis, and all inclusions are of spheroidal shape and arranged in such way that their symmetric axes are in parallel with the x_3-axis, the composite is also transversely isotropic with the x_3-axis to be the symmetric axis. there are only 10 independent constants.

THE EFFECTIVE CONSTANTS OF PIEZOELECTRIC COMPOSITES WITH A NON-DILUTE DISTRIBUTION OF INCLUSIONS

For the composite system with a non-dilute distribution of inclusions, we have to consider the interaction among inclusions. In general, it is impossible to obtain the average field inside the inclusions, or in the matrix. Therefore, the following approximation approach is developed.

For constant boundary conditions as shown in equation (30), the average strain $<\varepsilon^M_{ij}>$ and the average electric field $<E^M_k>$ in the matrix also keep constant. To consider the interaction among the inclusions, it is assumed that every single inclusion is subjected to

$<\varepsilon^M_{ij}>$, $<E^M_k>$ instead of ε^0_{ij} and E^0_k,

which is the same assumption as in the Mori-Tanaka approach. Thus with the aid of equations (2) and (3), the strain and electric field inside an inclusion can be expressed in the form as

$$\varepsilon^I_{ij} = B^1_{ijkl}<\varepsilon^M_{kl}> + B^2_{ijk}<E^M_k> \quad (34)$$

$$E^I_i = B^3_{ijk}<\varepsilon^M_{jk}> + B^4_{ik}<E^M_k> \quad (35)$$

The other two relations between the average field inside inclusions and the average field in the matrix are

$$\varepsilon^0_{ij} = v_f<\varepsilon^I_{ij}> + (1 - v_f)<\varepsilon^M_{ij}> \quad (36)$$

$$E^0_k = v_f<E^I_k> + (1 - v_f)<E^M_k> \quad (37)$$

Combining equatons (34) and (35) with equations (36) and (37), one obtains the average field in the matrix and inside the inclusions

$$\begin{bmatrix} <\varepsilon^M_{11}> \\ <\varepsilon^M_{22}> \\ <\varepsilon^M_{33}> \\ <\varepsilon^M_{23}> \\ <\varepsilon^M_{13}> \\ <\varepsilon^M_{12}> \end{bmatrix} = \begin{bmatrix} M^1_{11} & M^1_{12} & M^1_{13} & 0 & 0 & 0 \\ M^1_{12} & M^1_{11} & M^1_{13} & 0 & 0 & 0 \\ M^1_{31} & M^1_{31} & M^1_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & M^1_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & M^1_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & M^1_{66} \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon^0_{11} \\ \varepsilon^0_{22} \\ \varepsilon^0_{33} \\ \varepsilon^0_{23} \\ \varepsilon^0_{13} \\ \varepsilon^0_{12} \end{bmatrix} + \begin{bmatrix} 0 & 0 & M^2_{13} \\ 0 & 0 & M^2_{13} \\ 0 & 0 & M^2_{33} \\ 0 & M^2_{42} & 0 \\ M^2_{42} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E^0_1 \\ E^0_2 \\ E^0_3 \end{bmatrix} \quad (38)$$

$$\begin{bmatrix} \langle E_1^M \rangle \\ \langle E_2^M \rangle \\ \langle E_3^M \rangle \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & M_{15}^3 & 0 \\ 0 & 0 & 0 & M_{15}^3 & 0 & 0 \\ M_{31}^3 & M_{31}^3 & M_{33}^3 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} \varepsilon_{11}^0 \\ \varepsilon_{22}^0 \\ \varepsilon_{33}^0 \\ \varepsilon_{23}^0 \\ \varepsilon_{13}^0 \\ \varepsilon_{12}^0 \end{bmatrix} + \begin{bmatrix} M_{11}^4 & 0 & 0 \\ 0 & M_{11}^4 & 0 \\ 0 & 0 & M_{33}^4 \end{bmatrix} \begin{bmatrix} E_1^0 \\ E_2^0 \\ E_3^0 \end{bmatrix} \quad (39)$$

where

$$\begin{bmatrix} M_{11}^1 & M_{12}^1 & M_{13}^1 \\ M_{12}^1 & M_{11}^1 & M_{13}^1 \\ M_{31}^1 & M_{31}^1 & M_{33}^1 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{12} & g_{11} & g_{13} \\ g_{31} & g_{31} & g_{33} \end{bmatrix}^{-1} \quad (40)$$

and

$$g_{11} = 1 - v_f + v_f B_{11}^1 - v_f^2 \Delta_1 B_{13}^2 B_{31}^3 \quad (41)$$

$$g_{33} = 1 - v_f + v_f B_{33}^1 - v_f^2 \Delta_1 B_{33}^2 B_{33}^3 \quad (42)$$

$$g_{12} = v_f B_{12}^1 - v_f^2 \Delta_1 B_{13}^2 B_{31}^3 \quad (43)$$

$$g_{13} = v_f B_{13}^1 - v_f^2 \Delta_1 B_{13}^2 B_{33}^3 \quad (44)$$

$$g_{31} = v_f B_{31}^1 - v_f^2 \Delta_1 B_{33}^2 B_{31}^3 \quad (45)$$

$$M_{44}^1 = \left[1 - v_f + v_f B_{44}^1 - \frac{1}{2} v_f^2 (1 - v_f + v_f B_{11}^4)^{-1} B_{42}^2 B_{15}^3\right]^{-1} \quad (46)$$

$$M_{66}^1 = \left[1 - v_f + v_f B_{66}^1\right]^{-1} \quad (47)$$

$$M_{13}^2 = -v_f \Delta_1 (M_{11}^1 B_{13}^2 + M_{12}^1 B_{13}^2 + M_{13}^1 B_{33}^2) \quad (48)$$

$$M_{33}^2 = -v_f \Delta_1 (M_{31}^1 B_{13}^2 + M_{31}^1 B_{13}^2 + M_{33}^1 B_{33}^2) \quad (49)$$

$$M_{42}^2 = -\frac{1}{2} v_f (1 - v_f + v_f B_{11}^4)^{-1} M_{44}^1 B_{42}^2 \quad (50)$$

$$M_{31}^3 = -v_f \Delta_1 (M_{11}^1 B_{31}^3 + M_{12}^1 B_{31}^3 + M_{13}^1 B_{33}^3) \quad (51)$$

$$M_{33}^3 = -v_f \Delta_1 (M_{13}^1 B_{31}^3 + M_{13}^1 B_{31}^3 + M_{33}^1 B_{33}^3) \quad (52)$$

$$M_{15}^3 = -v_f (1 - v_f + v_f B_{11}^4)^{-1} B_{15}^3 M_{44}^1 \quad (53)$$

$$M_{11}^4 = (1 - v_f + v_f B_{11}^4)^{-1} + \frac{1}{2} v_f^2 (1 - v_f + v_f B_{11}^4)^{-2} B_{15}^3 B_{42}^2 M_{44}^1 \quad (54)$$

$$M_{33}^4 = \Delta_1 - v_f \Delta_1 (B_{31}^3 M_{13}^2 + B_{31}^3 M_{13}^2 + B_{33}^3 M_{33}^2) \quad (55)$$

where

$$\Delta_1 = (1 - v_f + v_f B_{11}^4)^{-1} \quad (56)$$

$$\begin{bmatrix} \langle \varepsilon_{11}^I \rangle \\ \langle \varepsilon_{22}^I \rangle \\ \langle \varepsilon_{33}^I \rangle \\ \langle \varepsilon_{23}^I \rangle \\ \langle \varepsilon_{13}^I \rangle \\ \langle \varepsilon_{12}^I \rangle \end{bmatrix} = \begin{bmatrix} I_{11}^1 & I_{12}^1 & I_{13}^1 & 0 & 0 & 0 \\ I_{12}^1 & I_{11}^1 & I_{13}^1 & 0 & 0 & 0 \\ I_{31}^1 & I_{31}^1 & I_{33}^1 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{44}^1 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_{44}^1 & 0 \\ 0 & 0 & 0 & 0 & 0 & I_{66}^1 \end{bmatrix} \times \begin{bmatrix} \varepsilon_{11}^0 \\ \varepsilon_{22}^0 \\ \varepsilon_{33}^0 \\ \varepsilon_{23}^0 \\ \varepsilon_{13}^0 \\ \varepsilon_{12}^0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & I_{13}^2 \\ 0 & 0 & I_{13}^2 \\ 0 & 0 & I_{33}^2 \\ 0 & I_{42}^2 & 0 \\ I_{42}^2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E_1^0 \\ E_2^0 \\ E_3^0 \end{bmatrix} \quad (57)$$

$$\begin{bmatrix} \langle E_1^I \rangle \\ \langle E_2^I \rangle \\ \langle E_3^I \rangle \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & I_{15}^3 & 0 \\ 0 & 0 & 0 & I_{15}^3 & 0 & 0 \\ I_{31}^3 & I_{31}^3 & I_{33}^3 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} \varepsilon_{11}^0 \\ \varepsilon_{22}^0 \\ \varepsilon_{33}^0 \\ \varepsilon_{23}^0 \\ \varepsilon_{13}^0 \\ \varepsilon_{12}^0 \end{bmatrix} + \begin{bmatrix} I_{11}^4 & 0 & 0 \\ 0 & I_{11}^4 & 0 \\ 0 & 0 & I_{33}^4 \end{bmatrix} \begin{bmatrix} E_1^0 \\ E_2^0 \\ E_3^0 \end{bmatrix} \quad (58)$$

where

$$I^1_{11}=B^1_{11}M^1_{11}+B^1_{12}M^1_{12}+B^1_{13}M^1_{31}+B^2_{13}M^3_{31} \quad (59)$$

$$I^1_{12}=B^1_{11}M^1_{12}+B^1_{12}M^1_{11}+B^1_{13}M^1_{31}+B^2_{13}M^3_{31} \quad (60)$$

$$I^1_{13}=B^1_{11}M^1_{13}+B^1_{12}M^1_{13}+B^1_{13}M^1_{33}+B^2_{13}M^3_{33} \quad (61)$$

$$I^1_{31}=B^1_{31}M^1_{11}+B^1_{31}M^1_{12}+B^1_{33}M^1_{31}+B^2_{33}M^3_{31} \quad (62)$$

$$I^1_{33}=B^1_{31}M^1_{13}+B^1_{31}M^1_{13}+B^1_{33}M^1_{33}+B^2_{33}M^3_{33} \quad (63)$$

$$I^1_{44}=\frac{1}{v_f}\left[1-(1-v_f)M^1_{44}\right] \quad (64)$$

$$I^1_{66}=\frac{1}{v_f}\left[1-(1-v_f)M^1_{66}\right] \quad (65)$$

$$I^2_{13}=B^1_{11}M^2_{13}+B^1_{12}M^2_{13}+B^1_{13}M^3_{33}+B^2_{13}M^4_{33} \quad (66)$$

$$I^2_{33}=B^1_{31}M^2_{13}+B^1_{31}M^2_{13}+B^1_{33}M^3_{33}+B^2_{33}M^4_{33} \quad (67)$$

$$I^2_{42}=-\frac{1}{v_f}(1-v_f)M^2_{42} \quad (68)$$

$$I^3_{31}=B^3_{31}M^1_{11}+B^3_{31}M^1_{12}+B^3_{33}M^1_{31}+B^4_{33}M^3_{31} \quad (69)$$

$$I^3_{33}=B^3_{31}M^1_{13}+B^3_{31}M^1_{13}+B^3_{33}M^1_{33}+B^4_{33}M^3_{33} \quad (70)$$

$$I^3_{15}=-\frac{1}{v_f}(1-v_f)M^3_{15} \quad (71)$$

$$I^4_{33}=B^3_{31}M^2_{13}+B^3_{31}M^2_{13}+B^3_{33}M^2_{33}+B^4_{33}M^4_{33} \quad (72)$$

$$I^4_{11}=\frac{1}{v_f}\left[1-(1-v_f)M^4_{11}\right] \quad (73)$$

Substitution of equations (57) and (58) into equations (28) and (29) yield the effective constants of a piezoelectric composite with a non-dilute distribution of inclusions. The 10 independent constants are

$$c^*_{11}=v_f(c_{11}I^1_{11}+c_{12}I^1_{12}+c_{13}I^1_{31}-e_{31}I^3_{31}) + (1-v_f)(c^0_{11}M^1_{11}+c^0_{12}M^1_{12}+c^0_{13}M^1_{31}-e^0_{31}M^3_{31}) \quad (74)$$

$$c^*_{33}=v_f(c_{13}I^1_{13}+c_{13}I^1_{13}+c_{33}I^1_{33}-e_{33}I^3_{33}) + (1-v_f)(c^0_{13}M^1_{13}+c^0_{13}M^1_{13}+c^0_{33}M^1_{33}-e^0_{33}M^3_{33}) \quad (75)$$

$$c^*_{12}=v_f(c_{11}I^1_{12}+c_{13}I^1_{11}+c_{13}I^1_{31}-e_{31}I^3_{31}) + (1-v_f)(c^0_{11}M^1_{12}+c^0_{12}M^1_{11}+c^0_{13}M^1_{31}-e^0_{31}M^3_{31}) \quad (76)$$

$$c^*_{13}=v_f(c_{11}I^1_{13}+c_{12}I^1_{13}+c_{13}I^1_{33}-e_{31}I^3_{33}) + (1-v_f)(c^0_{11}M^1_{13}+c^0_{12}M^1_{13}+c^0_{13}M^1_{33}-e^0_{31}M^3_{33}) \quad (77)$$

$$c^*_{44}=c_{44}-(1-v_f)(c_{44}-c^0_{44})M^1_{44} + \frac{1}{2}(1-v_f)(e_{24}-e^0_{24})M^3_{15} \quad (78)$$

$$e^*_{31}=-v_f(c_{11}I^2_{13}+c_{12}I^2_{13}+c_{13}I^2_{33}-e_{31}I^4_{33}) - (1-v_f)(c^0_{11}M^2_{13}+c^0_{12}M^2_{13}+c^0_{13}M^2_{33}-e^0_{31}M^4_{33}) \quad (79)$$

$$e^*_{33}=-v_f(c_{13}I^2_{13}+c_{13}I^2_{13}+c_{33}I^2_{33}-e_{33}I^4_{33}) - (1-v_f)(c^0_{13}M^2_{13}+c^0_{13}M^2_{13}+c^0_{33}M^2_{33}-e^0_{33}M^4_{33}) \quad (80)$$

$$c^*_{44}=c_{44}-(1-v_f)(c_{44}-c^0_{44})M^1_{44} \quad (81)$$

$$\alpha^*_{11}=\alpha_{11}-(1-v_f)(\alpha_{11}-\alpha^0_{11})M^4_{11} - 2(1-v_f)(e_{24}-e^0_{24})M^2_{42} \quad (82)$$

$$\alpha^*_{33}=v_f(e_{31}I^2_{13}+e_{31}I^2_{13}+e_{33}I^2_{33}-\alpha_{33}I^4_{33}) + (1-v_f)(e^0_{31}M^2_{13}+e^0_{31}M^2_{13}+e^0_{33}M^2_{33}-\alpha^0_{33}M^4_{33}) \quad (83)$$

NUMERICAL RESULTS AND DISCUSSIONS

As an example, piezoceramic polymer composite is used in this paper. PZT ceramic, which is commonly used as a piezoelectric transducer material, suffers from several disadvantages when used as a hydrostatic pressure sensor. The hydrostatic piezoelectric coefficient e^0_h ($= e^0_{33} + 2e^0_{31}$) of PZT is low due to opposite signs of piezoelectric coefficients e^0_{33} and e^0_{31}, even though the magnitudes of both constants are large. The hydrostatic voltage coefficient g^0_h ($= e^0_h/\alpha^0\varepsilon_0$) is also small because of its high relative permittivity α^0. ε_0 is the permittivity of free space. To improve the magnitude of e_h^0 and g_h^0, a number of different diphasic composites using PZT and passive polymers have been fabricated. It was shown that the phase connectivity of PZT ceramic and the polymer is the key feature in designing the composite materials. The material constants depend strongly on the phase connectivity. In a diphasic composite, there are ten possible connectivity patterns designated as 0-0, 1-0, 2-0, 3-0, 1-1, 2-1, 3-1, 2-2, and 3-3. The model developed in this paper can be used for all these materials. Here we only take the 3-0 composite as an example. A composite with 3-0 connectivity consists of a three-dimensionally connected PZT ceramic mixed with polymer particles. The material properties are as follows.

Table1. Material properties of PZT and polymer

	Elastic					Piezoelectri		Dielectric	
	c_{11}	c_{33}	c_{44}	c_{12}	c_{13}	e_{31} e_{15}	e_{33}	α_{11} α_{33}	
PZT	16.8	16.3	2.71	6.0	6.0	-2.9 4.6	7.1	36 34	
Pol	0.45	0.45	0.11	0.24	0.24	0 0	0	0 0	0

By using the equations obtained for the composite with a non-dilute distribution of inclusions, the numerical calculations were carried out. The results of composite properties as the functions of volume fraction and the aspect ratio of polymer inclusions are shown in Fig.1 - Fig.11.

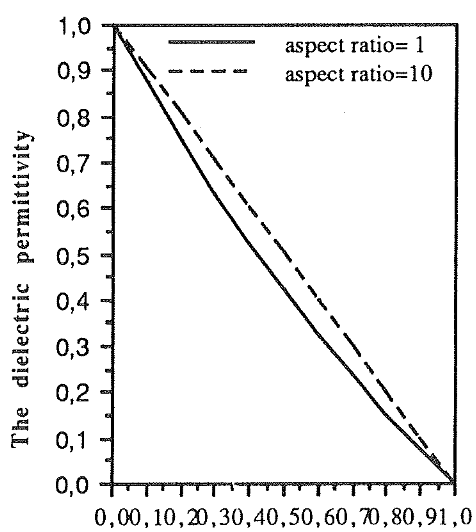

Fig.1. The dielectric permittivity versus the volume fraction of polymer inclusions

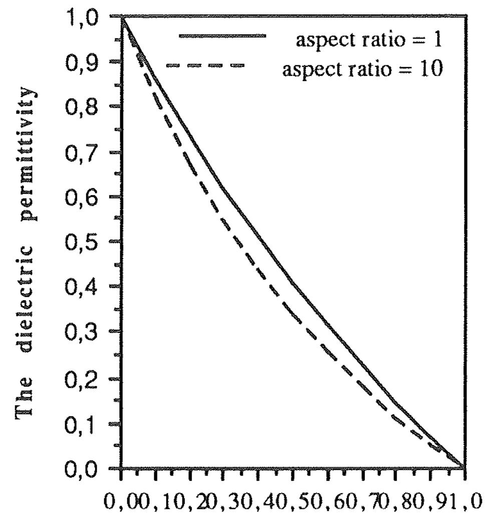

Fig.2. The dielectric permittivity versus the volume fraction of polymer inclusions

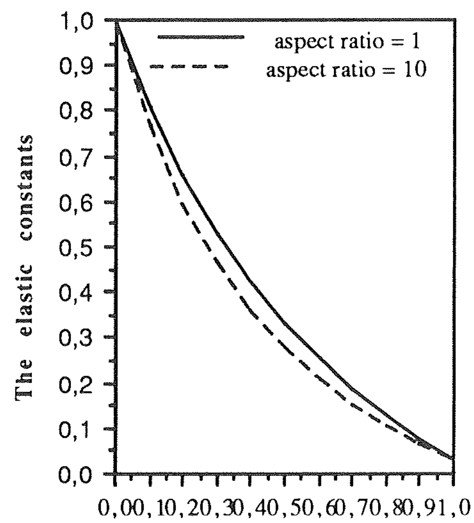

Fig. 3. The elastic constants versus the volume fraction of polymer inclusions

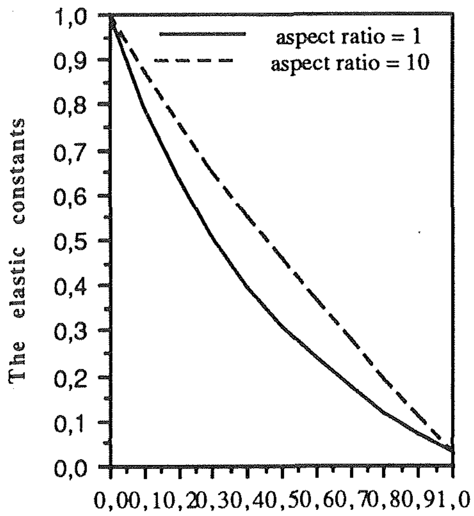

Fig. 4. The elastic constants versus the volume fraction of polymer inclusions

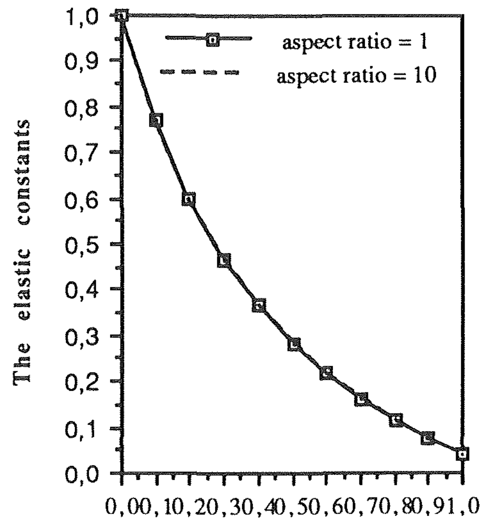

Fig. 6. The elastic constants versus the volume fraction of polymer inclusions

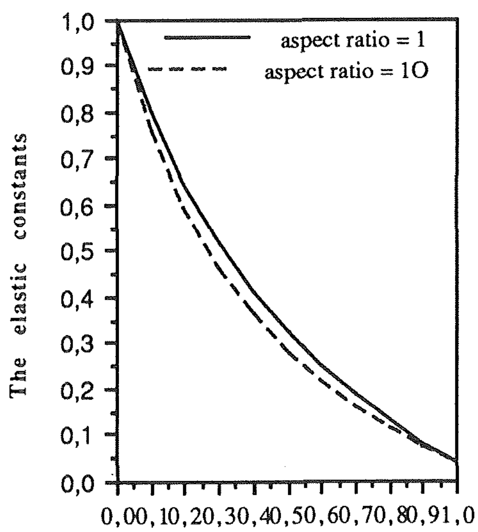

Fig. 5. The elastic constants versus the volume fraction of polymer inclusions

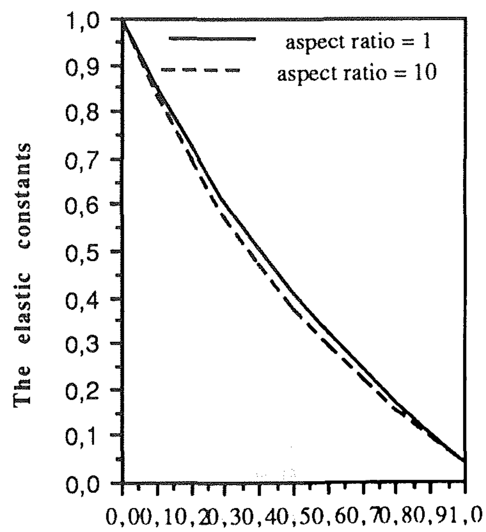

Fig. 7. The elastic constants versus the volume fraction of polymer inclusions

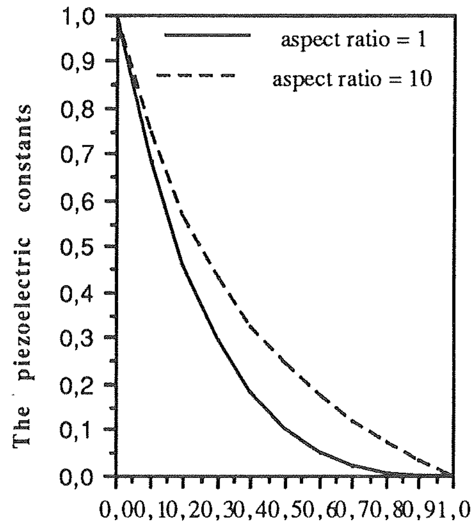

Fig. 8. The piezoelectric constants versus the volume fraction of polymer inclusions

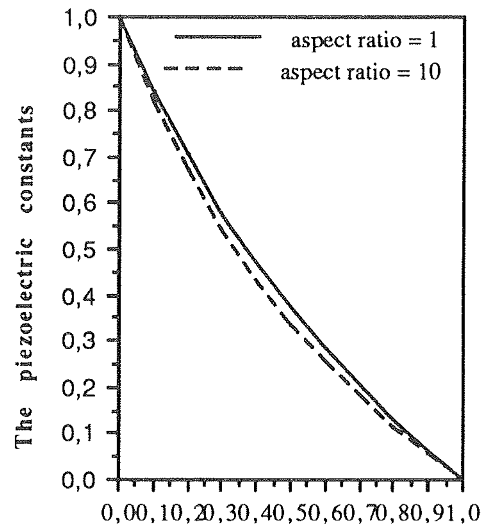

Fig. 10. The piezoelectric constants versus the volume fraction of polymer inclusions

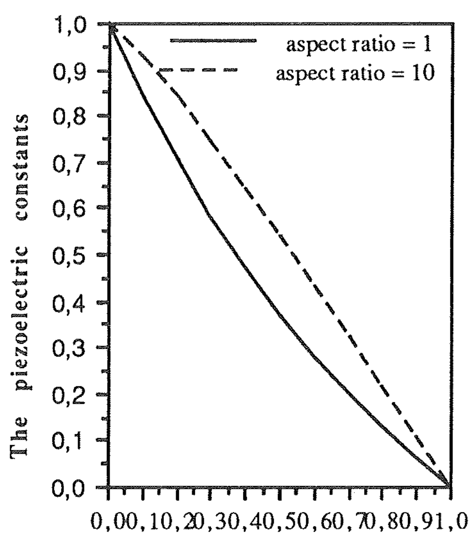

Fig. 9. The piezoelectric constants versus the volume fraction of polymer inclusions

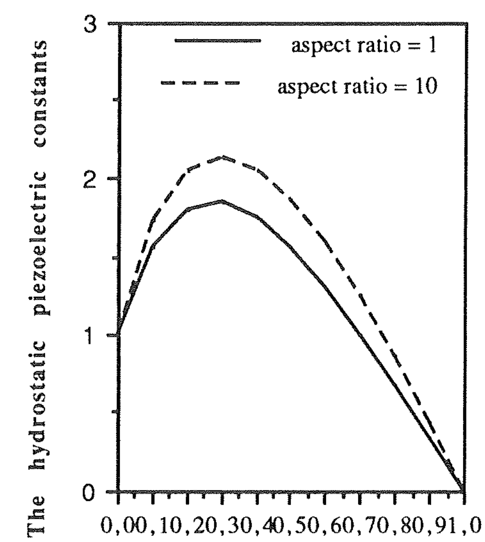

Fig.11. The hydrostatic piezoelectric constants versus the volume fraction of polymer inclusions

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APPENDIX: THE COMPONENTS OF $G^1_{ij}, G^2_{ij}, G^3_{ij}$

$$G^1_{12} = N^1_{1212} \tag{A1}$$

$$G^1_{13} = N^1_{1313} \tag{A2}$$

$$G^1_{44} = \frac{1}{4}(2N^1_{1313} + N^1_{1133} + N^1_{3311}) \tag{A3}$$

$$G^1_{66} = \frac{1}{2}(N^1_{1212} + N^1_{1122}) \tag{A4}$$

$$G^2_{13} = N^2_{113} \tag{A5}$$

$$G^2_{24} = \frac{1}{2}(N^2_{311} + N^2_{113}) \tag{A6}$$

and

$$\begin{gathered}\Delta^{-1} = [\beta^2(1-\omega^2)c^0_{66} + c^0_{44}\omega^2]\,\{[\beta^2(1-\omega^2)\alpha^0_{11} + \alpha^0_{33}\omega^2] \\ \{[\beta^2(1-\omega^2)c^0_{11} + c^0_{44}\omega^2][\beta^2(1-\omega^2)c^0_{44} + c^0_{33}\omega^2] - \beta^2\omega^2(1-\omega^2)(c^0_{13} + c^0_{44})^2\} \\ + [\beta^2(1-\omega^2)c^0_{11} + c^0_{44}\omega^2][\beta^2(1-\omega^2)e^0_{15} + e^0_{33}\omega^2]^2 \\ - 2\beta^2\omega^2(1-\omega^2)(c^0_{13} + c^0_{44})(e^0_{31} + e^0_{15})[\beta^2(1-\omega^2)e^0_{15} + e^0_{33}\omega^2] \\ + \beta^2\omega^2(1-\omega^2)(e^0_{31} + e^0_{15})^2[\beta^2(1-\omega^2)c^0_{44} + c^0_{33}\omega^2]\}\end{gathered} \tag{A7}$$

$$\begin{gathered}G^1_{11} = \frac{\pi}{2}\beta^2\int_0^1 (1-\omega^2)\Delta\{\{[\beta^2(1-\omega^2)c^0_{44} + c^0_{33}\omega^2] \\ [\beta^2(1-\omega^2)(c^0_{11} + 3c^0_{66}) + 4c^0_{44}\omega^2] - \beta^2\omega^2(1-\omega^2)(c^0_{13} + c^0_{44})^2\} \\ [\beta^2(1-\omega^2)\alpha^0_{11} + \alpha^0_{33}\omega^2] + [\beta^2(1-\omega^2)(c^0_{11} + 3c^0_{66}) + 4c^0_{44}\omega^2] \\ [\beta^2(1-\omega^2)e^0_{15} + e^0_{33}\omega^2]^2 - \beta^2\omega^2(1-\omega^2)(e^0_{31} + e^0_{15}) \\ [\beta^2(1-\omega^2)(2e^0_{15}c^0_{13} + e^0_{15}c^0_{44} - e^0_{31}c^0_{44}) + \\ \omega^2(2e^0_{33}c^0_{13} + 2e^0_{33}c^0_{44} - e^0_{31}c^0_{33} - e^0_{15}c^0_{33})]\}\, d\omega\end{gathered} \tag{A8}$$

$$\begin{gathered}G^1_{33} = 4\pi\int_0^1 \omega^2\Delta\{[\beta^2(1-\omega^2)c^0_{66} + c^0_{44}\omega^2][\beta^2(1-\omega^2)c^0_{11} + c^0_{44}\omega^2] \\ [\beta^2(1-\omega^2)\alpha^0_{11} + \alpha^0_{33}\omega^2] + \beta^2\omega^2(1-\omega^2)(e^0_{31} + e^0_{15})^2 \\ [\beta^2(1-\omega^2)c^0_{66} + c^0_{44}\omega^2]\}\, d\omega\end{gathered} \tag{A9}$$

$$\begin{gathered}N^1_{1122} = \frac{\pi}{2}\beta^2\int_0^1 (1-\omega^2)\Delta\{\{[\beta^2(1-\omega^2)(3c^0_{11} + c^0_{66}) + 4c^0_{44}\omega^2] \\ [\beta^2(1-\omega^2)c^0_{44} + c^0_{33}\omega^2] - 3\beta^2\omega^2(1-\omega^2)(c^0_{13} + c^0_{44})^2\} \\ [\beta^2(1-\omega^2)\alpha^0_{11} + \alpha^0_{33}\omega^2] - 3\beta^2\omega^2(1-\omega^2)(e^0_{31} + e^0_{15}) \\ [\beta^2(1-\omega^2)(2e^0_{15}c^0_{13} + e^0_{15}c^0_{44} - e^0_{31}c^0_{44}) \\ + \omega^2(2e^0_{33}c^0_{13} + 2e^0_{33}c^0_{44} - e^0_{31}c^0_{33} - e^0_{15}c^0_{33})] \\ + [\beta^2(1-\omega^2)(3c^0_{11} + c^0_{66}) + 4c^0_{44}\omega^2][\beta^2(1-\omega^2)e^0_{15} + e^0_{33}\omega^2]^2\}\, d\omega\end{gathered} \tag{A10}$$

$$\begin{gathered}N^1_{1133} = 2\pi\int_0^1 \omega^2\Delta\{\{[\beta^2(1-\omega^2)(c^0_{11} + c^0_{66}) + 2c^0_{44}\omega^2] \\ [\beta^2(1-\omega^2)c^0_{44} + c^0_{33}\omega^2] - \beta^2\omega^2(1-\omega^2)(c^0_{13} + c^0_{44})^2\} \\ [\beta^2(1-\omega^2)\alpha^0_{11} + \alpha^0_{33}\omega^2] - \beta^2\omega^2(1-\omega^2)(e^0_{31} + e^0_{15}) \\ [\beta^2(1-\omega^2)(2e^0_{15}c^0_{13} + e^0_{15}c^0_{44} - e^0_{31}c^0_{44}) \\ + \omega^2(2e^0_{33}c^0_{13} + 2e^0_{33}c^0_{44} - e^0_{31}c^0_{33} - e^0_{15}c^0_{33})] \\ + [\beta^2(1-\omega^2)(c^0_{11} + c^0_{66}) + 2c^0_{44}\omega^2][\beta^2(1-\omega^2)e^0_{15} + e^0_{33}\omega^2]^2\}\, d\omega\end{gathered} \tag{A11}$$

$$\begin{gathered}N^1_{3311} = 2\pi\beta^2\int_0^1 (1-\omega^2)\Delta\{[\beta^2(1-\omega^2)c^0_{66} + c^0_{44}\omega^2] \\ [\beta^2(1-\omega^2)c^0_{11} + c^0_{44}\omega^2][\beta^2(1-\omega^2)\alpha^0_{11} + \alpha^0_{33}\omega^2] \\ + \beta^2\omega^2(1-\omega^2)(e^0_{31} + e^0_{15})^2[\beta^2(1-\omega^2)c^0_{66} + c^0_{44}\omega^2]\}\, d\omega\end{gathered} \tag{A12}$$

$$\begin{gathered}N^1_{1212} = \frac{\pi}{2}\beta^4\int_0^1 (1-\omega^2)^2\Delta\{[\beta^2(1-\omega^2)(2c^0_{13}e^0_{15} + c^0_{44}e^0_{15} - c^0_{44}e^0_{31}) \\ + \omega^2(2e^0_{33}c^0_{13} + 2e^0_{33}c^0_{44} - e^0_{31}c^0_{33} - e^0_{15}c^0_{33})]\,\omega^2(e^0_{15} + e^0_{31}) \\ + \{\omega^2(c^0_{13} + c^0_{44})^2 - (c^0_{11} - c^0_{66})[\beta^2(1-\omega^2)c^0_{44} + c^0_{33}\omega^2]\} \\ [\beta^2(1-\omega^2)\alpha^0_{11} + \alpha^0_{33}\omega^2] - (c^0_{11} - c^0_{66})[\beta^2(1-\omega^2)e^0_{15} + e^0_{33}\omega^2]^2\}\, d\omega\end{gathered} \tag{A13}$$

$$\begin{gathered}N^1_{1313} = -2\pi\beta^2\int_0^1 \omega^2(1-\omega^2)\Delta[\beta^2(1-\omega^2)c^0_{66} + c^0_{44}\omega^2] \\ \{(c^0_{13} + c^0_{44})[\beta^2(1-\omega^2)\alpha^0_{11} + \alpha^0_{33}\omega^2] + (e^0_{31} + e^0_{15}) \\ [\beta^2(1-\omega^2)e^0_{15} + e^0_{33}\omega^2]\}\, d\omega\end{gathered} \tag{A14}$$

$$\begin{gathered}N^2_{113} = 2\pi\beta^2\int_0^1 \omega^2(1-\omega^2)\Delta[\beta^2(1-\omega^2)c^0_{66} + c^0_{44}\omega^2] \\ [\beta^2(1-\omega^2)(e^0_{31}c^0_{44} - e^0_{15}c^0_{13}) + \omega^2(e^0_{31}c^0_{33} + e^0_{15}c^0_{33} \\ - e^0_{33}c^0_{13} - e^0_{33}c^0_{44})]\, d\omega\end{gathered} \tag{A15}$$

$$\begin{gathered}N^2_{311} = 2\pi\beta^2\int_0^1 (1-\omega^2)\Delta\{[\beta^2(1-\omega^2)c^0_{66} + c^0_{44}\omega^2] \\ [\beta^2(1-\omega^2)c^0_{11} + c^0_{44}\omega^2][\beta^2(1-\omega^2)e^0_{15} + e^0_{33}\omega^2] \\ - \beta^2\omega^2(1-\omega^2)(e^0_{31} + e^0_{15})(c^0_{13} + c^0_{44})[\beta^2(1-\omega^2)c^0_{66} + c^0_{44}\omega^2]\}\, d\omega\end{gathered} \tag{A16}$$

$$\begin{gathered}G^2_{33} = 4\pi\int_0^1 \omega^2\Delta\{[\beta^2(1-\omega^2)c^0_{66} + c^0_{44}\omega^2] \\ [\beta^2(1-\omega^2)c^0_{11} + c^0_{44}\omega^2][\beta^2(1-\omega^2)e^0_{15} + e^0_{33}\omega^2] \\ - \beta^2\omega^2(1-\omega^2)(e^0_{31} + e^0_{15})(c^0_{13} + c^0_{44})[\beta^2(1-\omega^2)c^0_{66} + c^0_{44}\omega^2]\}\, d\omega\end{gathered} \tag{A17}$$

$$\begin{gathered}G^3_{11} = 2\pi\beta^2\int_0^1 (1-\omega^2)\Delta\{\beta^2\omega^2(1-\omega^2)[\beta^2(1-\omega^2)c^0_{66} + c^0_{44}\omega^2] \\ (c^0_{13} + c^0_{44})^2 - [\beta^2(1-\omega^2)c^0_{66} + c^0_{44}\omega^2] \\ [\beta^2(1-\omega^2)c^0_{11} + c^0_{44}\omega^2][\beta^2(1-\omega^2)c^0_{44} + c^0_{33}\omega^2]\}\, d\omega\end{gathered} \tag{A18}$$

$$\begin{gathered}G^3_{33} = 4\pi\int_0^1 \omega^2\Delta\{\beta^2\omega^2(1-\omega^2)[\beta^2(1-\omega^2)c^0_{66} + c^0_{44}\omega^2] \\ (c^0_{13} + c^0_{44})^2 - [\beta^2(1-\omega^2)c^0_{66} + c^0_{44}\omega^2] \\ [\beta^2(1-\omega^2)c^0_{11} + c^0_{44}\omega^2][\beta^2(1-\omega^2)c^0_{44} + c^0_{33}\omega^2]\}\, d\omega\end{gathered} \tag{A19}$$